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Reg. No. : ....

Name : ....

Fifth Semester B.Sc. Degree Examination, December 2021

First Degree Programme under CBCSS

Mathematics

Core Course V

MM 1541 : REAL ANALYSIS – I

(2014, 2016 & 2017 Admission)

Time : 3 Hours Max. Marks : 80

SECTION – I

All the **first 10** questions are compulsory. They carry **1** mark each.

1. Find all real numbers x that satisfy the inequality $\frac{1}{x} < x$.

2. Let E consist of all numbers $1/n$, where n= 1,2, 3....find inf E.

3. State the Supremum property of R.

4. Find the cluster points of the set $E = \{1/n : n \in N\}$.

5. What is a Fibonacci sequence

6. Define limit of a sequence.

7. Give an example of a sequence which is bounded but not convergent.

8. What is the 'sum' of an infinite series?

P.T.O.

9. What is $\lim_{x \to 0} \frac{x^2}{|x|}$?

10. Show that the sequence 1, 1/2, 1, 1/3, 1, 1/4... diverges

SECTION – II

Answer **any eight** questions (11-22). Each questions carries **2** marks.

11. State the order complete property of R.

12. State the Dedekind's theorem.

13. State and prove the Archimedean property of R.

14. State and prove Bernoulli's inequality.

15. Give examples to show that the supremum and infimum of a set may or may not belong to the set.

16. Show by an example that every bounded real sequence may not be convergent.

17. Check whether the sequence $\{n + (-1)^n\}$ is monotonic or not.

18. Show that a convergent sequence of real numbers is bounded.

19. Show that $\lim_{n \to \infty} \frac{\cos n}{n} = 0$,

20. Find $\lim_{x \to 0} \frac{1 - \cos x}{x}$.

21. Show that $\sum_{1}^{\infty} \frac{1}{\sqrt{n^2 + 1}}$ is divergent.

22. Show that if $\sum x_n$ convergence, then $\lim_{x \to 0} x_n = 0$.

SECTION – III

Answer **any six** questions (23-31). Each questions carries **4** marks.

23. Prove that between any two real numbers there is a rational one.

24. Suppose that $\{a_n\}$, $\{b_n\}$ and $\{c_n\}$ are sequences such that $a_n \le b_n \le c_n$ for all $n \ge 10$ and that $\{a_n\}$ and $\{c_n\}$ both converge to 12. Then show that $\{b_n\}$ also converges to 12.

25. Show that $\sum_{1}^{\infty} \sin\frac{1}{n}$ is divergent.

26. Show that the sequence $\left\{\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \ldots, \frac{n}{n+1} \ldots\right\}$.

27. If $\{a_n\}$ converges to l, show that every subsequence of $\{a_n\}$ converges to l.

28. Show that $\sum_{1}^{\infty} \frac{n}{n^p}$ converges if $p > 1$.

29. Discuss the convergence of the series whose n^{th} term is $(10n+1)/(n(n+1)(n+2))$.

30. Test the series whose general term is given by $U_n = \left(1 - \frac{1}{n}\right)n^2$ is convergent.

31. Give an example of a series which is convergent but not absolutely .

SECTION – IV

Answer **any two** questions (32-35). Each questions carries **15** marks.

32. (a) State and prove nested interval property of real numbers.

    (b) Show that there exists no rational number whose square is 2.

33. Let $\{x_n\}$ and $\{y_n\}$ be two sequences of real numbers such that $\lim x_n = x$ and $\lim y_n = y$ and c is a real number. Show that

(a) $\lim(x_n + y_n) = \lim x_n + \lim y_n = x + y$

(b) $\lim(x_n - y_n) = \lim x_n - \lim y_n = x - y$

(c) $\lim(x_n y_n) = \lim x_n \cdot \lim y_n = xy$ and

(d) $\lim(c\, x_n) = c \lim x_n = c_x$

34. (a) State and prove Bolzano Weierstrass theorem.

(b) Define a contractive sequence and show that every contractive sequence is Cauchy.

35. (a) A sequence is convergent if and only if it is a Cauchy sequence.

(b) Establish the sequential criterion for a limit of a function.