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Reg. No. : ....

Name : ....

Fifth Semester B.Sc. Degree Examination, December 2021

First Degree Programme Under CBCSS

Mathematics

Core Course V

MM 1542 – COMPLEX ANALYSIS – I

(2014, 2016 & 2017 Admission)

Time : 3 Hours Max. Marks : 80

SECTION – I

All the first **ten** questions are compulsory. They carry **1** mark each.

1. Write the polar form of $-1-i$.
2. If w is a non real cube root of unity then $1+w+w^2=$ ——————.
3. Define a power series in z.
4. Find $\{z : |z+2i| < 1\}$.
5. Define a region.
6. What is real part of $\frac{5+\sqrt{2i}}{1-\sqrt{2i}}$.
7. State closed curve theorem.
8. When we say $f(z)$ is differentiable at a point z?

P.T.O.

9. $\log(-i) =$ ______________.

10. Solve : $z^4 - 1 = 0$.

SECTION – II

Answer **any eight** questions from among the questions **11** to **22**. These questions carry **2** marks each.

11. Find the multiplicative inverse of the complex number $a + ib$.

12. Prove that if a product if two complex number is zero, then at least one of them is zero.

13. Explain iz geometrically.

14. Prove $|z_1 - z_2| \geq ||z_1| - |z_2||$.

15. Find the equation of the circle with centre at $2 + 3i$ and passing through $1 + i$.

16. Find all values of k such that $f(z) = e^x(\cos ky + i \sin ky)$ is analytic.

17. If z is a product of two complex numbers a and b then prove that argument of z is sum of argument of a and argument of b.

18. Prove that $\sin 3\theta = 4 \sin^3 \theta - 3 \sin \theta$.

19. Solve $z^4 + 5z^2 = 36$.

20. Evaluate $\int_c z^2$ where c is the straight line joining origin to the point $2 + i$.

21. Define the derivative of a complex valued function $f(z)$ and prove that $f(z) = \bar{z}$ is not differentiable.

22. If f and g are both differentiable at z then prove that $f + g$ and $f \cdot g$ are also differentiable.

SECTION – III

Answer **any six** from among the questions **23** to **31**. These questions carry **4** marks each.

23. If $f(z) = \sum_{n=0}^{\infty} C_n z^n$ has a non zero radius of convergence, then prove that $C_n = \frac{f^{(n)}(0)}{n!}$ for all n.

24. If $x + iy = \frac{a + ib}{a - ib}$ then prove that $x^2 + y^2 = 1$.

25. If $f(z) = u + iv$ is analytic in a region D and u is constant then prove that f is constant.

26. If $f(z) = x^2 + iy^2$ and $C : z = t + it, 0 \leq t \leq 1$ then find $\int_C f(z)\, dz$.

27. Find all analytic functions $f(z) = u + iv$ where $u = x^2 - y^2$.

28. If C is given by $z(t)$; $a \leq t \leq b$ then prove that $\int_{-C} f = -\int_C f$.

29. If f is a linear function and if Γ is the boundary of a rectangle, then prove that $\int_\Gamma f(z)\, dz = 0$.

30. Determine the convergence of the series on the circle of convergence.

    (a) $\sum_{n=0}^{\infty} z^n$

    (b) $\sum_{0}^{\infty} \frac{z}{n}$.

31. Find the real part of $\tan^{-1}(x + iy)$.

SECTION – IV

Answer **any two** questions from among the questions **32** to **35**. These questions carry **15** marks each.

32. If $\tan x = \frac{1}{2}$ find the value of $\tan 5x$.

33. Find the set of points in the complex plane satisfying the conditions

(a) $\left|\frac{z+1}{z-1}\right| = k(\text{constant})$

(b) $\arg\left(\frac{z-1}{z+2}\right) = \frac{\pi}{3}$.

34. (a) Convert $\frac{-16}{1+i\sqrt{3}}$ into polar form.

(b) Suppose $G(t)$ is a continuous complex valued function of f , then prove that $\int_a^b G(t)\,dt << \int_a^b |G(t)|\,dt$ and using it prove the M-L formula.

35. (a) If $u - v = (x - y)\left(x^2 + 4xy + y^2\right)$ then find u and v separately.

(b) If $f(z)$ is analytic function of z then prove that $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\left(|f(z)|^2\right) = 4\ |f'(z)|^2$.
