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Reg. No. : ....

Name : ....

Fifth Semester B.Sc. Degree Examination, December 2021

First Degree Programme Under CBCSS

Mathematics

Core Course

MM 1544 — VECTOR ANALYSIS

(2014, 2016 & 2017 Admission)

Time : 3 Hours Max. Marks : 80

SECTION – A

All the **first ten** questions are compulsory. **Each** carries **1** mark.

1. Find the gradient of $f(x,y) = x^2 + y$ at (1, 1).
2. If $r = xi + yj + zk$, show that $\nabla\|r\| = \dfrac{r}{\|r\|}$.
3. Define the divergence of a vector point functions and find the divergence of $F = e^x(\cos yi + \sin yj)$.
4. Define line integral of a continuous vector field along a smooth curve.
5. Define the potential function for a vector field.
6. Is the field $F = 2xy^3 i + (1 + 3x^2 y^2) j$ is conservative. Justify?
7. If C is the curve represented by the equation $x = 1 + 2t$, $y = t$, $-1 \le t \le 1$, evaluate $\int_C \dfrac{x}{1+y^2}\, ds$.

P.T.O.

8. Find the parametric representation of the cone $z = \sqrt{x^2 + y^2}$, $0 \le z \le \pi$.

9. State fundamental theorem on line integrals.

10. State Stoke's theorem.

SECTION – B

Answer **any eight** questions from this section. **Each** question carries **2** marks.

11. Find the directional derivative of $f(x,y) = xe^y + \cos(xy)$ at the point (2, 0) in the direction of $v = 3i - 4j$.

12. Prove that for any smooth scalar point function $\phi = \phi(x,y,z), curl(\nabla\phi) = 0$.

13. Evaluate the line integral $\int_C (x - y)ds$, *where* C is the helix given by $x = 2t$, $y = 3t^3$, $(0 \le t \le 1)$.

14. Find the work done by the force $F(x,y) = x^3yi + (x - y)j$ on a particle which moves along the curve $y = x^2$ from (–2, 4) to (1,1).

15. Find the area of the surface that extends upwards from the parabola $y = x^2$, $(0 \le x \le 2)$ in the *xy*-plane to the plane $z = 3x$.

16. Evaluate the line integral $\int_C (x^2ydx + xdy)$ using greens theorem, where C is the triangular path with vertices (0, 0), (1, 0) and (1, 2).

17. Using line integral to find the area enclosed by the astroid $x = a\cos^3\phi$, $y = a\sin^3\phi$, $0 \le \phi \le 2\pi$.

18. Calculate the outward flux of the vector field $F(x,y) = xi + y^2j$ across the square bounded by the lines $x = \pm 1$ and $y = \pm 1$.

19. Evaluate $\int_C 4xy\,dx + 2xy\,dy$ where C is the rectangle bounded by the lines $x = -2x = 4, y = 1$ and $y = 2$.

20. Find the mass of the wire that lies along the curve $r(t) = (t^2 - 1)i + 2tk$, $0 \le t \le 1$, if the density $\delta = \frac{3}{2}t$.

21. For what values of b and c will $F = (y^2 + 2czx)i + y(bx + cz)j + (y^2 + cx^2)k$ be a gradient field.

22. Find the outward flux of the vector field $F(x, y, z) = 2xi + 3yj + z^2k$ across the unit cube bounded by the planes $x = 0$, $y = 0$, $z = 0$, $x = 1$, $y = 1$ and $z = 1$.

SECTION – C

Answer **any six** questions from among the questions **23** to **31**. These questions carry **4** marks each.

23. Let $f(x, y) = \frac{x^2}{2} + \frac{y^2}{2}$. Find the maximum and minimum value of a directional derivative at (1, 1), and find those unit vectors in the direction in which the maximum and minimum values occurs.

24. Show that the inverse square filed $F(x, y, z) = \frac{C}{(x^2 + y^2 + z^2)^{\frac{3}{2}}}(xi + yj + zk)$ is conservative.

25. Show that the integral $\int_{(1,2)}^{(4,0)} 4dy + 4xdy$ is independent of path and find its value.

26. If ϕ is a scalar point function so that $grad(\phi(x, y, z)) = yzi + zxj + xyk$, find ϕ if ϕ(1, 1, 1) = 1.

27. Show that $F = (z\cos x + \sin y)i + (x\cos y + \sin z)j + (y\cos z + \sin x)k$ is conservative and find its potential function.

28. Using Greens theorem find the work done by the force field $F = -y^2i + xyj$ along the curve which is the boundary of the square cut from the first quadrant by the lines x = 1 and y = 1.

29. Integrate $G(x,y,z) = x^2$ over the cone $z = \sqrt{x^2 + y^2}, 0 \le z \le 1$.

30. Verify Gauss divergence theorem for $F = (x^2 - yz)i + (y^2 - zx)j + (z^2 - xy)k$, over the rectangular parallelopiped $0 \le x \le a$, $0 \le y \le b$, $0 \le z \le c$.

31. Find the flux of $F = x^2 i + 3y^2 j$ across S, where S is the portion of the plane $x + y + z = 1$ in the first octant.

SECTION – D

Answer **any two** questions from among the questions **32** to **35**. These questions carry **15** marks **each**.

32. (a) Prove that $div(\phi F) = \phi(div F) + \nabla\phi . F$ or any differentiable vector function F and for any differentiable scalar point function ϕ.

(b) Let $r = xi + yj + zk$ and $r = |r|$. Then prove that if *f* is a differentiable function of *r,* then $div(r\, f(r)) = r\, f'(r) + 3f(r)$.

33. (a) State stokes theorem and verify it for the field $F = [y, z, x]$ for the paraboloid S given by $z = 1 - (x^2 + y^2)$, $z \ge 0$.

(b) Evaluate the surface integral $\iint_{\sigma} xz dS$ where σ is the part of the plane $x + y + z = 1$ that lies in the first octant.

34. Find the flux of $F = yzi + xj - z^2 k$ through the parabolic cylinder $y = x^2$, $0 \le x \le 1$, $0 \le z \le 4$ in the direction of out normal vector.

35. Verify divergence theorem for the vector field $F = xi + yj + zk$ over the sphere $x^2 + y^2 + z^2 = a^2$.