**(Pages : 4)** **M – 1459**

**Reg. No. : .....**

**Name : .....**

**Fifth Semester B.Sc. Degree Examination, December 2021.**

**First Degree Programme under CBCSS**

**Mathematics**

**Core Course VIII**

**MM 1544 – DIFFERENTIAL EQUATIONS**

**(2018 & 2019 Admission)**

Time : 3 Hours Max. Marks : 80

SECTION – I

Answer **all** questions :

1. Write the standard equation of linear differential equation.
2. Write the Lipschitz condition.
3. Solve $dy + y\,dx = 0$.
4. For what values of the constant $m$ will $y = e^{mx}$ be the solution of $y''-3y' - 10y = 0$.
5. Check whether $y^2dy + x^2dx$ exact or not.
6. Find the complementary function of $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = e^x \sin x$. .
7. Define Wronskian
8. Write the standard form of Legendre's linear equation

**P.T.O.**

9. Write the characteristic equation of $2\frac{d^2y}{dx^2} - \frac{dy}{dx} - 3y = 0$.

10. Define basis of solutions of a homogeneous second order differential equation

**($10 \times 1 = 10$ Marks)**

SECTION – II

Answer **any eight** questions

11. Find the order and degree of the ODE $\frac{d^3y}{dx^3} + 2\left(\frac{dy}{dx}\right)^{\frac{1}{2}} = 0$

12. Define partial differential equation. Give one example of it

13. Solve $\frac{dy}{dx} = xy + x$.

14. State the uniqueness theorem of first order differential equation.

15. Verify that $y = \frac{2}{x}$ is a solution of the differential equation $xy' = -y$ ,for all $x \neq 0$.

16. Show that a seperable equation is also exact.

17. Check the exactness of $y' = 1 + y^2$.

18. Find the integrating factor of $y\,dx - x\,dy = 0$.

19. Find the general solution of $\frac{d^2y}{dx^2} + 4y = 0$.

20. Find a differential equation whose solution is cos 3x.

21. Find the complementary function of $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = 3e^x$.

22. Write the basis of solution of the equation $\frac{d^2y}{dx^2} + y = 0$

23. Write the standard form of Euler- Cauchy equation. Give one example of it.

24. Solve $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2 = 0$

25. Find a general solution of $x^2y' - 20y = 0$.

26. Find the Wronskian of $e^x$ and $e^{-x}$.

**($8 \times 2 = 16$ Marks)**

SECTION – III

Answer **any six** questions

27. Solve $(3x^2 + 4xy)dx + (2x^2 + 2y)dy = 0$

28. Solve $(x + 4)(y^2 + 1)dx + y(x^2 + 3x + 2)dy = 0$

29. Find the Orthogonal Trajectories of the family $cx^2 + y^2 = 1$

30. Solve the initial value problem $y' + y\tan x = \sin 2x, y(0) = 1$

31. Solve $x\frac{dy}{dx} + y = xy^{\frac{3}{2}}, y(1) = 4$.

32. Solve $(x^2 - 3y^2)dx + 2xy\,dy = 0$

33. By reducing the order, solve $(x^2 + 1)y''-2xy' + 2y = 0$, given $x$ is one solution

34. Solve $\frac{d^2y}{dx^2} + y = \sin x$.

35. Find the general solution of the equation $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 6e^x$

36. Solve $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} - 16y = 0$.

37. Solve the logistic equation $y' = Ay - By^2$.

38. Solve $y''+y = \cos ec\, x$ using the method of variation of parameters.

**($6 \times 4 = 24$ Marks)**

SECTION – IV

Answer **any two** questions

39. (a) Solve $\left(\frac{3-y}{x^2}\right)dx+\left(\frac{y^2-2x}{xy^2}\right)dy=0,\ y(-1)=2$ by exactness.

(b) Find an integrating factor and solve

$(5xy+4y^2+1)dx+(x^2+2xy)dy=0$

40. (a) Solve the initial value problem $(ye^x+2e^x+y^2)dx+(e^x+2xy)dy=0,$ $y(0)=6$

(b) Find a basis of solutions of the differential equation $(x^2-x)y''-xy'+y=0$.

41. (a) Check the exactness and solve $(2xy^2+y)dx+(2y^3-x)dy=0$.

(b) Solve the initial value problem $(y+\sqrt{x^2+y^2})dx-xdy=0,\ y(1)=0$.

42. (a) Solve $x^2y''-2xy+2y=0,\ y(1)=1,\ y'(1)=1.$

(b) Solve $\frac{d^2y}{dx^2}-3\frac{dy}{dx}+2y=2x^2+e^x+2xe^x+4e^{3x}$.

43. (a) Solve $(D^2+2D+\frac{3}{4}I)y=3e^x+\frac{9}{2}x$.

(b) Solve $y'''-3y''+2y'=0$.

44. (a) Solve the initial value problem

$y''-2y'-3y=2e^x-10\sin x,\ y(0)=2,\ y'(0)=4$

(b) Solve $(D^2+3D+2I)y=5x^2$.

**($2\times 15 = 30$ Marks)**

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