(Pages : 4) **P – 2472**

Reg. No. : ....

Name : ....

Fifth Semester B.Sc. Degree Examination, December 2022

First Degree Programme under CBCSS

Mathematics

Core Course – V

MM 1542 : COMPLEX ANALYSIS

(2013 Admission)

Time : 3 Hours Max. Marks : 80

SECTION – I

All the first **ten** questions are compulsory. They carry **1** mark each.

1. Define a domain in the complex plane.

2. Give an example of an open set in the complex plane.

3. Define a bounded set in the complex plane.

4. Write an example of an entire function.

5. Write Cauchy-Riemann equations in polar form.

6. Define the circle of convergence of a power series.

7. Define a harmonic function.

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8. Write down the power series whose sum function is $f(z) = \frac{1}{e^z}$.

9. Define the cross ratio of the four points $z,\ z_1,\ z_2, z_3$.

10. Is the mapping $w = \bar{z}$ conformal?

(10 × 1 = 10 Marks)

SECTION – II

Answer any **eight** questions. Each question carries **2** marks.

11. Sketch the set S of points in the complex plane satisfying the inequality $|z - 3| < 3$ and determine whether the set is a domain.

12. Determine whether $\lim_{z \to 0} \frac{z}{|z|}$ exists or not.

13. Prove that every differentiable function is continuous.

14. Determine the singular points of the function $\frac{1}{(z - 1)(z + 3)}$.

15. Check whether the function $f(z) = z + \bar{z}$ is analytic or not.

16. Show that $f(z) = e^x(\cos y + i \sin y)$ is analytic.

17. Prove that an analytic function with a constant real part is a constant.

18. Determine the constants a, b, c if the function $f(z) = x + ay + i(2x + by)$ is analytic.

19. Find the Taylor series expansion of the function $f(z) = \sin z$ about $z = \pi / 4$.

20. Find the fixed points of the transformation $w = \frac{z - 1}{z + 1}$.

21. Find the points in the z-plane at which the mapping $f(z) = z^5 - 5z$ is not conformal.

22. Find the image of the quadrant $x > 1, y > 0$ under the mapping $w = \frac{1}{z}$.

(8 × 2 = 16 Marks)

SECTION – III

Answer any **six** questions. Each question carries **4** marks.

23. Show that an analytic function is independent of $\bar{z}$.

24. Show that if $f(z)$ is analytic in a domain D and $|f(z)|$ is a constant in D, then $f(z)$ is a constant in D.

25. Suppose $f(z) = u(x, y) + i\,v(x, y)$ is analytic in a domain D, prove that $u(x, y) = c_1$ and $v(x, y) = c_2$ form an orthogonal system of curves.

26. Prove that $f(z) = \frac{1}{z^2}$ is nowhere analytic.

27. Expand $f(z) = |z|^2$ as a Taylor series about $z = 2$. State the region of validity.

28. Find the Laurent's series expansion for the function $f(z) = \frac{1}{z(z-3)}$ valid for $0 < |z| < 3$.

29. Prove that a cross ratio is invariant under linear fractional transformation.

30. Find the bilinear transformation that maps $0, -i, -1$ onto i, 1, 0.

31. Prove that the transformation $w = f(z) = \frac{1}{z}$ maps a circle in z - plane to a circle in w-plane or a straight line in w – plane.

(6 × 4 = 24 Marks)

SECTION – IV

Answer any **two** questions. **Each** question carries **15** marks.

32. Derive Cauchy — Riemann equations in Cartesian form

33. (a) Show that the function $f(z) = e^{y}e^{ix}$ is nowhere analytic.

    (b) Show that the function $f(z) = \frac{x^3(1+i) - y^3(1-i)}{(x^2 + y^2)}, z \neq 0$ and $f(0) = 0$ satisfies Cauchy - Riemann equations at z = 0 but not differentiable at z = 0.

34. (a) Show that the real and imaginary parts of an analytic function are harmonic.

    (b) Check whether $u(x, y) = x^3 - 3xy^2 + 3x^2 - 3y^2 + 1$ is harmonic. If so, find the harmonic conjugate of u and the corresponding analytic unction in terms of z.

35. (a) Find the image of the strip $1 < y < 2$ under the transformation w = sin z.

    (b) Show that the transformation $w = \frac{i - z}{1 + z}$ maps the real axis of the z - plane onto the circle $|w| = 1$ and the half plane $y > 0$ onto the interior of the unit circle $|w| = 1$ in the w-plane.

(2×15 = 30 Marks)