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Reg. No. : ....

Name : ....

Fifth Semester B.Sc. Degree Examination, December 2022

First Degree Programme Under CBCSS

Mathematics

Core Course VII

MM1544 : NUMERICAL METHODS

(2013 Admission)

Time : 3 Hours Max. Marks : 80

SECTION – I

Answer **all** questions. **Each** question carries **1** mark.

1. Define absolute and relative errors.

2. Find an initial approximation to a root of the equation $2x^3 - x^2 + 2x - 1 = 0$, using location of roots theorem.

3. What is complete pivoting.

4. State Picard's n^{th} approximations to the solution of the initial value problem

5. State the Simpson's 3/8 rule for numerical integration.

6. Write down the fourth order Runge-Kutta formula for solving a first order differential equation.

7. What is interpolation

8. State the relation between backward difference operator and shift operator.

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9. State Stirling's formula

10. Give the general quadrature formula for numerical integration

(10 $\times$ 1 = 10 Marks)

SECTION – II

Answer **any eight** questions. **Each** question carries **2** marks.

11. If actual value is 5.5 *feet* and estimated value is 6 *feet* find the percentage error.

12. Perform three iterations of the bisection method to obtain the smallest positive root of the equation $x^3 - 5x + 1 = 0$.

13. Using Newton-Raphson method, find the iterative formula to calculate the square root of *N*.

14. Explain Gauss elimination method of solving a system of linear equations

15. Obtain Picard's second approximate solution of the initial value problem $\frac{dy}{dx} = x - y,\ y(0) = 0$

16. Explain Euler's method of solving a initial value problem $\frac{dy}{dx} = f(x, y), y(x_0) = y_0$.

17. Construct the forward difference table for the following set of (x, y) values (0,7), (5,11), (10,14), (15,18), (20,24) and (25,32).

18. Prove that $\delta \equiv \nabla E^{1/2}$.

19. Find a cubic polynomial which takes the following values: y(0) = 1,y(1) = 2, y(2) = 3, y(3) = 10.

20. Use Lagrange's interpolation formula to fit a polynomial to the data

x	0	1	3	4
y	–12	0	6	2

21. From the following data obtain the first derivative of $y = \log_e x$ at $x = 500$

x	500	510	520	530	540	550
$y = \log_e x$	6.2146	6.2344	6.2538	6.2729	6.2916	6.3099

22. Evaluate $\int_0^1 x^3 dx$ by Trapezoidal rule using 6 coordinates.

(8 × 2 = 16 Marks)

SECTION – III

Answer **any six** questions. **Each** question carries **4** marks.

23. Obtain a polynomial approximation $P(x)$ to e^{-x}, using Taylor series expansion about $x_0 = 0$. Determine the maximum error for $x \in [-1, 1]$ when the first four terms are used in the approximation.

24. Solve the equation $x^3 - 9x + 1 = 0$ for the root lying between 2 and 3, correct to three decimal places, using bisection method.

25. Prove that Newton-Raphson process has a quadratic convergence

26. A root of the equation $xe^x - 1 = 0$ lies in the interval (0.5,1.0). Determine this root correct to three decimal places using regula-falsi method.

27. Use Gauss elimination method to solve the system: $3x + y - z = 3; 2x, -8y + z = -5; x - 2y + 9z = 8$.

28. Perform three iterations of the Gauss-Seidel method for solving the system of equations: $4x + 2z = 6; 5y + 2z = -3; 5x + 4y + 10z = 11$.

29. Obtain the approximate value of $y(0.2)$ for the initial $\frac{dy}{dx} = x^2 + y^2, y(0) = 1$ with $h = 0.1$, by using modified Euler's method.

30. If $y(75) = 246, y(80) = 202, y(85) = 118, y(90) = 40,$ find $y(79)$.

31. The following table gives values of y for different values of x. Find the value of x correct to one decimal place for which $y = 7$.

x	1	3	4
y	4	12	9

(6 × 4 = 24 Marks)

SECTION – IV

Answer **any two** questions. **Each** question carries **15** marks.

32. (a) Derive Newton-Raphson formula to calculate a root of the equation $f(x) = 0$. **7**

(b) Find the root of the equation $\sin x = 1 + x^3$, between –2 and –1 correct to 3 decimal places by Newton-Raphson method. **8**

33. (a) Solve $x + 2y + z - w = -2$; $2x + 3y - z + 2w = 7$; $x + y + 3z - 2w = 16$; $x + y + z + w = 2$ by Gauss-Jordan method. **7**

(b) Solve $8x - 3y + 2z = 20$; $6x + 3y + 12z = 35$; $4x + 11y - z = 33$ by Jacobi's iteration method. **8**

34. (a) Using Taylor's series method, solve $\frac{dy}{dx} = x^2 - y, y(0) = 1$ at $x = 0.1$ and $x = 0.2$. **7**

(b) Solve the initial value problem $\frac{dy}{dx} = x(y - x), y(2) = 3$ in the interval $[2, 2.4]$, using Runge-Kutta method of fourth order with the step size $h = 0.2$. **8**

35. (a) The following data gives the melting point of an alloy of zinc and lead, θ is the temperature and x is the percentage of lead . Using Newton's interpolation formula, find θ when $x = 48$ and $x = 84$. **8**

x	40	50	60	70	80	90
θ	184	204	226	250	276	304

(b) From the following table, find x, correct to two decimal places, for which y is maximum and find this value of y. **7**

x	1.2	1.3	1.4	1.5	1.6
y	0.9320	0.9636	0.9855	0.9975	0.9996

(2 × 15 = 30 Marks)