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Reg. No. : ....

Name : ....

Fifth Semester B.Sc. Degree Examination, December 2022

First Degree Programme under CBCSS

Mathematics

Core Course

MM 1542 : COMPLEX ANALYSIS I

(2014-2017 Admissions)

Time : 3 Hours Max. Marks : 80

PART – A

Answer **all** questions. Each carries **1** mark.

1. Find the square root of z_i.

2. Sketch the set of points $\{z \mid z = \bar{z}\}$.

3. Define an open set.

4. Define an entire function.

5. Show that $e^{2\pm 3\pi i} = -e^2$.

6. Find the derivative of $\sin z$.

7. Define a smooth curve.

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8. Evaluate $\int_0^1 (4t + it^2)\,dt$.

9. State the rectangle theorem.

10. Find $\int_c e^z dz$, where $c : |z| = 1$.

(10 $\times$ 1 = 10 Marks)

PART – B

Answer **any eight** questions. Each carries **2** marks.

11. Define a Cauchy sequence.

12. Check the convergence of $\sum_{k=1}^{\infty} \frac{i^k}{k^2 + 1}$.

13. Prove $\overline{z_1 + z_2} = \overline{z_1} + \overline{z_2}$.

14. Show that a nonconstant analytic polynomial cannot be real valued.

15. Find the radius of convergence of the series $\sum_{n=1}^{\infty} \frac{z^n}{n^2}$.

16. Prove that if $u + iv$ is analytic in a region D and u is constant, then and is constant.

17. Show that $\left| e^z \right| = e^x$.

18. Verify the identity : $\sin 2z = 2 \sin z \cos z$.

19. Show that $f(z) = |z|^2$ is nowhere analytic.

20. Define a complex line integral.

21. Let C be a smooth curve. Let f and g be continuous functions on C. Prove that

$$\int_{c} (f(2)+g(2)dz = \int_{c} f(z)\, dz + \int_{c} g(z) dz$$

22. Write the formula for the arc length of the curve

$$c : z(t) = x(t) + i\, y(t),\, a \le t \le b$$

(8 × 2 = 16 Marks)

PART – C

Answer **any six** questions. Each carries **4** marks.

23. Show that any quadratic equation with complex coefficients admits a solution in the complex field.

24. Prove that $\{z_n\}$ converges if and only if $\{z_n\}$ is a Cauchy sequence.

25. Prove that if $f(2) = \sum_{n=0}^{\infty} c_n z^n$ has nonzero radius of convergence, then $c_n = \frac{f^n(0)}{n!}$ for all n.

26. Show that $f(z) = \begin{cases} \frac{(\bar{z})^2}{z}, & \text{when } z \neq 0 \\ 0, & \text{when } z = 0 \end{cases}$ satisfies Cauchy Riemann equations at $z = 0$, but $f(z)$ is not differentiable at $z = 0$.

27. Find all solutions of $e^z = 3 + 4i$.

28. Find a power series representation for $\sin z$.

29. Prove that if f is a linear function and if Γ is the boundary of a rectangle R, then $\int_{\Gamma} f(z)\, dz = 0$.

30. Evaluate $\int_c f(z)dz$, where $f(z) = x^2 + 3ixy$ and c is the line segment joining $1+i$ to $2-i$.

31. Evaluate $\int_c \frac{1}{z}dz$, where $c : |z| = R, R \neq 0$.

(6 × 4 = 24 Marks)

PART – D

Answer **any two** questions. Each carries **15** marks.

32. (a) Prove that a region S is polygonally connected.

(b) Prove that a polynomial $p(x, y)$ is analytic if and only if $p_y = ip_x$.

33. (a) State and prove the uniqueness theorem for power series.

(b) If $\sum a_n z^n$ and $\sum b_n z^n$ converge and agree on a set of points with an accumulation point at the origin, prove that $a_n = b_n$ for all n.

34. (a) Suppose that g is the inverse of f at z_0 and that g is continuous there. Prove that if f is differentiable at $g(z_0)$ and if $f'(g(z_0)) = 0$, then g is differentiable at z_0 and $g'(z_0) = \frac{1}{f'(z_0)}$.

(b) Find all analytic functions $f = u + iv$ with $u(x, y) = 4xy - x^3 + 3xy^2$.

35. Prove that if c_1 and c_2 are smoothly equivalent, then $\int_{c_1} f = \int_{c_2} f$.

(2 × 15 = 30 Marks)

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