(Pages : 4) **P – 2481**

Reg. No. : ....

Name : ....

Fifth Semester B.Sc. Degree Examination, December 2022

First Degree Programme under CBCSS

Mathematics

Core Course

MM 1543 – DIFFERENTIAL EQUATIONS

(2014–2017 Admission)

Time : 3 Hours Max. Marks : 80

SECTION – I

All the first **ten** questions are compulsory. They carry **1** mark each.

1. Suppose that a particle moves along an s-axis in such a way that its velocity $v(t)$ is always half of $s(t)$. Find a differential equation whose solution is $s(t)$.

2. Define order of a differential equation.

3. Define an integral curve.

4. Verify whether $y = 2e^{x^3/3}$ is the solution of the initial value problem $y' = x^2y$, $y(0) = 2$.

5. Evaluate $\int \frac{\cos x}{\sin^2 x} dx$.

6. Give an example of a second order homogeneous linear differential equation.

P.T.O.

7. Give the general form of a homogeneous linear equation.

8. Define wronskian of two solutions y_1 and y_2.

9. Show that $y = 1$ is a solution of $y''y - xy' = 0$.

10. Define singular solution of a differential equation.

(10 × 1 = 10 Marks)

SECTION – II

Answer any **eight** questions. Each question carries **2** marks.

11. If 100 grams of radioactive carbon-14 are stored in a cave for 1000 years how many grams will be left at that time?

12. Solve the initial value problem $\frac{dy}{dx} = \sqrt[3]{x},\ y(1) = 2$.

13. Find an equation of the curve that passes through the point (–3,0) and the slope $2x + 1$.

14. Solve the differential equation $\frac{dy}{dx} + 3y = 0$.

15. Find the solution of the differential equation $x\frac{dy}{dx} + y = x$ that satisfies the initial condition $y(1) = 2$.

16. Find an exponential growth model $y = y_0 e^{kt}$ that satisfies $y(1) = 1$ and $y(0) = 100$.

17. Find the characteristic equation of the differential equation $y'' - y = 0$ and hence solve.

18. Show that e^x and xe^x are linearly independent on any interval.

19. Solve $x^2 y'' + 7xy' + 13y = 0$.

20. Solve the initial value problem $y'' + 4y' + 4y = 0,\ y(0) = 1,\ y'(0) = 1$.

21. Find the general solution of $y'' + \omega^2 y = 0$.

22. Solve the nonhomogeneous equation $y'' + 4y = 8x^2$.

(8 × 2 = 16 Marks)

SECTION – III

Answer any **six** questions. Each question carries **4** marks.

23. Suppose that a point moves along a curve $y = f(x)$ in the *xy*-plane in such a way that at each point (*x*, *y*) on the curve the tangent line has slope $-\sin x$. Find an equation for the curve given that it passes through the point (0, 2).

24. Solve the initial value problem $(4y - \cos y)\dfrac{dy}{dx} - 3x^2 = 0, y(0) = 0$.

25. Find a curve in the *xy*-plane that passes through (0, 3) and whose tangent line at a point (*x*, *y*) has slope $\dfrac{2x}{y^2}$.

26. Solve the differential equation $9y'' + 12y' + 29y = 0$.

27. Show that e^x is an integrating factor and solve $\sin y\, dx + \cos y\, dy = 0$.

28. Solve the initial value problem $y'' + 0.2y' + 4.01,\ y = 0,\ y'(0) = 2$.

29. Solve the Euler – Cauchy differential equation $x^2y'' - 3xy' + 3y = 0$.

30. Solve the initial value problem $y'' + y = 0,\ y(0) = 0$ and $y'(0) = 3$.

31. Solve the boundary value problem $y'' + 4y = 0,\ y(0) = 3,\ y\left(\dfrac{\pi}{2}\right) = -3$.

(6 × 4 = 24 Marks)

SECTION – IV

Answer any **two** questions. Each question carry **15** marks.

32. Suppose that the growth of a population $y = y(t)$ is given by the logistic equation $y = \dfrac{60}{5 + 7e^{-t}}$.

    (a) What is the population at time $t = 0$?

    (b) What is the carrying capacity L?

    (c) What is the constant k ?

    (d) Find an initial value problem whose solution is $y(t)$.

33. (a) Solve the initial value problem $x\dfrac{dy}{dx} - y = x$, $y(1) = 2$.

    (b) Solve the initial value problem $\dfrac{dy}{dx} = \dfrac{e^{2x}}{e^{y}}$ with $y(2) = 4$.

34. (a) Solve the differential equation $y'' + y = \sec x$.

    (b) Solve $y'' - 3y' + 2y = e^{x}$.

35. Find the steady – state oscillation of the mass-spring system governed by the equation $y'' + 3y' + 2y = 20\cos 2t$.

(2 × 15 = 30 Marks)
