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Reg. No. : ....

Name : ....

Fifth Semester B.Sc. Degree Examination, December 2022

First Degree Programme under CBCSS

Mathematics

Core Course

MM 1544 : VECTOR ANALYSIS

(2014 – 2017 Admission)

Time : 3 Hours Max. Marks : 80

SECTION – I

Answer **all** questions from this section. Each question carries 1 mark.

1. Define the divergence of $\overline{F}$.

2. Find the directional derivative of $f(x,y,z) = x^2y - yz^3 + z$ at (1, –2, 0) in the direction of the vector $2\hat{i} + \hat{j} - 2\hat{k}$.

3. Find the gradient of $\overline{F}(x,y,z) = e^z - \ln(x^2 + y^2)$.

4. State Gauss theorem.

5. Define curl of a vector field.

6. Find the divergence of $\overline{F} = 6x^2z\hat{i} + 2x^2y\hat{j} - yz^2\hat{k}$.

7. Show that $\overline{F} = yz\hat{i} + xz\hat{j} + xy\hat{k}$ is solenoidal.

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8. A vector that is perpendicular to both the vectors $\vec{A}$ and $\vec{B}$ is ————.

9. Calculate the curl of the vector field $\overline{F}$, $\overline{F}(x,y)=(x^2-2y)\hat{i}+(xy-y^2)\hat{j}$.

10. Div $(\overline{F}\times\overline{G})=$ ————.

(10 × 1 = 10 Marks)

SECTION – II

Answer any **eight** from the following. Each question carries **2** marks.

11. Find the curl $\overline{F}, \overline{F}=x^2z\hat{i}-2y^3z^2\hat{j}+xy^2z\hat{k}$ at (1, 1, 1).

12. Find the directional derivative of $f(x, y, z)=x^3-xy^2-z$ at (1, 1, 0) in the direction of $A=2\hat{i}-3\hat{j}+6\hat{k}$.

13. Define inverse square field. State Gauss law of inverse square field.

14. Show that $\overline{F}=(2x-3)\hat{i}-z\hat{j}+(\cos z)\hat{k}$ is not conservative.

15. Prove that if ϕ is a scalar function $curl\,(grad\ \phi)=0$.

16. Let $\vec{F}(x,y)=2x\hat{i}+3y\hat{j}+4z\hat{k}$. Find the potential function.

17. Evaluate $\oint_C (x-y)dx+x\,dy$, where C is the unit circle $\overline{r}(t)=\cos t\hat{i}+\sin t\hat{j}$, $0\le t\le 2\pi$.

18. Find the gradient field of $f(x, y, z)=(x^2+y^2+z^2)^{-1/2}$.

19. Evaluate $\int_C (xy+y+z)ds$ along the curve $\overline{r}(t)=2t\hat{i}+t\hat{j}+(2-2t)\hat{k}$ $0\le t\le 1$.

20. Find the workdone by $\overline{F}=(y-x^2)\hat{i}+(z-y^2)\hat{j}+(x-z^2)\hat{k}$ over the curve $\overline{r}(t)=t\hat{i}+t^2\hat{j}+t^3\hat{k}$, $0\le t\le 1$, from (0, 0, 0) to (1, 1, 1).

21. Find the circulation of the field $\overline{F}=(x-y)\hat{i}+x\,\hat{j}$ around the circle $x^2+y^2=1$.

22. Prove that $div\,(curl\,\overline{F})=0$.

(8 × 2 = 16 Marks)

SECTION – III

Answer any **six** from the following. Each question carries **4** marks.

23. Find the area enclosed by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

24. A fluids velocity field is $\overline{F} = x\hat{i} + z\hat{j} + y\hat{k}$. Find the flow along the helix $\overline{r}(t) = \cos t\hat{i} + \sin t\hat{j} + t\hat{k}$, $0 \le t \le \frac{\pi}{2}$.

25. Evaluate $\oint_C x^2 y\, dx + (y + xy^2)dy$, where C is the region enclosed by $y = x^2$, $x = y^2$.

26. Find the potential function of $\overline{F} = x^2 y\hat{i} + 5xy^2\hat{j}$.

27. Evaluate $\iint_S (7x\hat{i} - z\hat{k}) \cdot \overline{n}\, d\sigma$ over the sphere $x^2 + y^2 + z^2 = 4$ using divergence theorem.

28. Evaluate $\int_C \overline{F} \cdot d\overline{r}$, where $F = (x^2 + y^2)\hat{i} - 2xy\hat{j}$, C is the rectangle in the XY – plane bounded by $x = 0$, $x = a$, $y = 0$, $y = a$.

29. Prove that $div\,(f\overline{V}) = f \cdot div\,\overline{V} + \overline{V} \cdot grad\, f$, where f is a scalar function.

30. Find the flux of $\overline{F} = xy\hat{i} + yz\hat{j} + xz\hat{k}$ outward through the surface of cube cut from first octant by planes $x = 1$, $y = 1$, $z = 1$.

31. If $\overline{F} = (3x^2 + 6y)\hat{i} - 14yz\hat{j} + 20xz^2\hat{k}$. Evaluate $\int_C \overline{F} \cdot d\overline{r}$, where C is the straight line joining (0, 0, 0) and (1, 0, 0).

(6 $\times$ 4 = 24 Marks)

SECTION – IV

Answer any **two** from the following. **Each** question carries **15** marks.

32. (a) State Stoke's theorem.

(b) Verify Stokes theorem for $F = y\hat{i} - x\hat{j}$ for the hemisphere $S : x^2 + y^2 + z^2 = 9$, $z \geq 0$, its bounding circle $C : x^2 + y^2 = 9,\ z = 0$.

33. Evaluate the surface integral $\iint_{\sigma} x^2\, ds$ over the sphere $x^2 + y^2 + z^2 = 1$.

34. Prove that

(a) $div(\phi\overline{F}) = grad\,\phi\overline{F} + \phi div\,\overline{F}$

(b) $curl(\phi\overline{F}) = (grad\,\phi)\times\overline{F} + \phi\ curl\,\overline{F}$

35. Verify divergence theorem for the function $\overline{F} = (x, y, z) = x\hat{i} + y\hat{j} + z\hat{k}$, where σ is the spherical surface $x^2 + y^2 + z^2 = 1$.

(2 $\times$ 15 = 30 Marks)