Reg. No. : ....

Name : ....

First Semester B.Sc. Degree Examination, June 2022

First Degree Programme under CBCSS

Core Course

Mathematics

MM 1141 : METHODS OF MATHEMATICS

(2020 Admission Onwards)

Time : 3 Hours Max. Marks : 80

SECTION – I

(Answer **all** questions Compulsory. Each question carries **1** marks.)

1. Find an equation for the local linear approximation to $y = 5 - x^2$ at $x_0 = 2$.

2. If f has an inflection point at $x = x_0$, then $f''(x_0) =$ —————.

3. State the extreme value Theorem.

4. The function $f(x) = |x| - 1$ has how many horizontal tangent to the graph of f over $(-1, 1)$.

5. If f is differentiable and $f'(x) \neq 0$ on (a, b), then the equation $f(x) = 0$.

6. Find the velocity and speed of the function $s(t) = t^3 - 6t^2$.

7. A cylindrical shell is enclosed by two concentric right circular cylinders. The volume of that cylindrical shell is —————.

8. The lateral area of the frustum with slant height $\sqrt{10}$ base radii $\dot{r}_1 = 1$ and $r_2 = 2$ is ——————.

9. The volume of the torus generated by revolving a circular region of radius b about a line at a distance a from the centre of the circle is ——————.

10. Find the volume of the solid that is obtained when the region under the curve $y = \sqrt{2}$ over the interval $[1,4]$ is revolved about the x-axis.

(10 × 1 = 10 Marks)

SECTION – II

(Answer **any eight** question. Each question carries **2** marks)

11. Express the derivative with respect to x of $y = x^2$ in differential form at $x = 1$.

12. Evaluate $\lim\limits_{x \to \frac{\pi}{2}} \dfrac{1 - \sin x}{\cos x}$ using *L*'Hospital's rule.

13. Find the inflection points, if any, of $f(x) = x^4$.

14. Find all critical points of $f(x) = 3x^{\frac{5}{3}} - 15x^{\frac{2}{3}}$.

15. Find the x-intercepts of the equation $y = x^3 - 3x + 2$.

16. Define Rolle's Theorem.

17. Find $\dfrac{dy}{dx}$ for $y = \dfrac{x^2 - 1}{x^3}$.

18. Find the absolute extrema of $f(x) = 6x^{\frac{4}{3}} - 3x^{\frac{1}{3}}$ in the interval $[-1, 1]$.

19. Find the volume of the solid that is obtained when the region under the curve $y = \sqrt{x}$ over the interval $[1,4]$ is resolved about the x-axis.

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20. A spring exerts a force of 5 *N* when stretched 1 *m* beyond its natural length. Find the spring constant *k.*

21. Find the fluid pressure and force on the top of a flat circular plate of radius 2 *m* that is submerged horizontally in water at a depth of 6 *m.*

22. Prove that $\cosh^2 x - \sinh^2 x = 1$.

23. Find the total mass of a triangular lamina with vertices (0,0), (0,1) and (1,0) has density $\delta = 3$.

24. Prove that $\dfrac{d}{dx}[\sinh x] = \cosh x$.

25. Find the value of $\displaystyle\int_{-2}^{3} (x(x+6) = x^2) \, dx.$

26. Define Hooke's Law.

(8 × 2 = 16 Marks)

SECTION – III

(Answer **any six** question. Each question carries **4** marks)

27. The diameter of a polyurethane sphere is measured with percentage error within ± 0.4%. Estimate the percentage error in the calculated volume of the sphere.

28. Find $\displaystyle\lim_{x \to 0} (1 + \sin x)^{\frac{1}{x}}$.

29. Find the relative extrema of $f(x) = 3x^5 - 5x^3$.

30. Determine whether the function $f(x) = \dfrac{1}{x^2 - x}$ has any absolute extrema on the interval (0, 1). If so, find them.

31. Explain the steps for solving Applied Maximum and Minimum Problems.

32. Find the absolute extrema, if any, of the function $f(x) = e^{(x^3 - 3x^2)}$ on the interval (0, +∞).

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33. Derive the formula for the volume of a right pyramid whose altitude is h and whose base is a square with sides of length a.

34. A space probe of mass in $m = 5.00 \times 10^4$ kg travels in deep space subjected only to the force of its own engine. Starting at a time when the speed of the probe is $v = 1.10 \times 10^4$ m/s, thee engine is fired continuously over a distance of 2.50×10^6 m with a constant force of 4.00×10^5 N in the direction of motion. What is the final speed of the probe.

35. Derive the formula for the volume of a sphere of radius r.

36. State and prove the Pappus Theorem.

37. Explain Fluid Pressure.

38. A liquid form of antibiotic manufactured by a pharmaceutical firm is sold in bulk at a price of \$200 per unit. If the total production cost for x units is $C(x) = 500,000 + 80x + 0.003x^2$.

(6 × 4 = 24 Marks)

SECTION – IV

(Answer **any two** question. Each question carries **15** marks)

39. Find a point on the curve $y = x^2$ that is closest to the point (18,0).

40. Find the area of the region enclosed by $x = y^2$ and $y = x - 2$.

41. Suppose that the position function of a particle moving on a coordinate line is given by $s(t) = 2t^3 - 21t^2 + 60t + 3$..

42. Explain the surface area problem.

43. Derive the problem for Fluid force on a vertical surface.

44. A 100 ft wire is attached at its ends to the tops of two 50 ft poles that are positioned 90 ft apart. How high above the ground is the middle of wire ?

(2 × 15 = 30 Marks)

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