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S – 1630

Reg. No. : ....

Name : ....

Fifth Semester B.Sc. Degree Examination, December 2023

First Degree Programme under CBCSS

Mathematics

Core Course V

MM 1541 : REAL ANALYSIS I

(2018 Admission Onwards)

Time : 3 Hours   Max. Marks : 80

SECTION – I

All the first **ten** questions are compulsory. They carry **1** mark each.

1. Write the greatest lower bound for a set $A \subseteq R$.

2. Define a one-to-one function.

3. Define a countable set

4. When will you say a Sequence Converges?

5. Define a Cauchy sequence.

6. When will you say a series converges absolutely?

7. State true or false: The sum of positive terms $\sum_{n=1}^{\infty} \frac{1}{2n-1}$ diverges to infinity.

P.T.O.

8. Define a compact set.

9. What is an F_σ set?

10. What is the basic example of a compact set?

(10 × 1 = 10 Marks)

SECTION – II

Answer any **eight** questions. These questions carry **2** marks each.

11. If $A \subseteq B$ and B is countable, then prove that A is either countable or finite.

12. Show that (0, 1) is uncountable if and only if **R** is uncountable.

13. Define the power set P(A) of a set A. Let A = {a, b, c}. List the eight elements of P(A).

14. Give an example of (a) Sequences (x_n) and (y_n), which both diverge, but whose sum $(x_n + y_n)$ converges, (b) A convergent sequence (b_n) with $b_n \neq 0$ for all $n \in N$ such that $(1/b_n)$ diverges.

15. Show that $\lim_{n \to \infty}\left(\sqrt{n+1} - \sqrt{n}\right) = 0$.

16. Let (a_n) and (b_n) be sequences of real numbers such that $\lim a_n = a$ and $\lim b_n = b$. Then show that $\lim(a_n + b_n) = a + b$.

17. Let (a_n) and (b_n) be sequences of real numbers such that $\lim_{n \to \infty} a_n = a$, and $\lim_{n \to \infty} b_n = b$. If $a_n \geq 0$ for all $n \in N$, then show that $a \geq 0$.

18. Let $(a_n) \to 0$ and use Algebraic limit theorem, compute $\lim_{n \to \infty} \dfrac{1 + 2a_n}{1 + 3a_n - 4a\dfrac{2}{n}}$

19. Define the closure of a set $A \subseteq R$. Find the closure of $\left\{\frac{1}{n} : n \in N\right\}$.

20. Define the Cantor set.

21. Let $A = \left\{(-1)^n + \frac{2}{n} : n = 1, 2, 3, \ldots\right\}$ and $B = \{x \in Q : 0 < x < 1\}$. Find the limit points of each set. Also find the closure of A and B.

22. Prove that $\{x \in R : c \leq x \leq d\}$ is a closed set.

(8 × 2 = 16 Marks)

SECTION – III

Answer any **six** questions. These questions carry **4** marks each.

23. State and prove the Nested Interval Property.

24. Show that Q is countable.

25. Assume $s \in \mathbf{R}$ is an upper bound for a set $A \subseteq \mathbf{R}$. Then show that s = sup A if and only if, for every choice of $\epsilon > 0$, there exists an element $a \in A$ satisfying $s - \epsilon < a$.

26. Prove that every convergent sequence is bounded.

27. Let (a_n) and (b_n) be sequences of real numbers such that $\lim a_n = a$ and $\lim b_n = b$. Then show that $\lim(a_n/b_n) = a/b$ provided $b \neq 0$.

28. State and prove Monotone Convergence Theorem.

29. Prove that a set $F \subseteq \mathbf{R}$ is closed if and only if every Cauchy sequence contained in F has a limit that is also an element of F.

30. Show that a set F is closed if and only if F^C is open.

31. Show that the union of a finite collection of closed sets is closed.

(6 × 4 = 24 Marks

SECTION – IV

Answer any **two** questions. These questions carry **15** marks each.

32. Show that there exists a real number $\alpha \in R$ satisfying $\alpha^2 = 2$.

33. Define a monotone sequence. Give an example. Also state and prove the Monotone Convergence Theorem.

34. If $K_1 \supseteq K_2 \supseteq K_3 \supseteq K_4 \supseteq \bullet\bullet\bullet$ is a nested sequence of nonempty compact sets, then prove that the intersection $\bigcap_{n=1}^{\infty} K_n$ is not empty.

35. State and prove the Heine – Borel Theorem.

(2 × 15 = 30 Marks)
