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Reg. No. : .....

Name : .....

First Semester B.Sc. Degree Examination, March 2023

First Degree Programme under CBCSS

Mathematics

Core Course

MM 1141 : METHODS OF MATHEMATICS

(2018 Admission Onwards)

Time : 3 Hours Max. Marks : 80

SECTION – I

All questions are compulsory. Each question carries **1** mark.

1. What is the local linear approximation of $f(x)=\sqrt{x}$ at $x_0=1$.

2. Define point of inflection.

3. Define critical point.

4. State Extreme value theorem.

5. For a particle in rectilinear motion, the acceleration and position functions $a(t)$ and $s(t)$ are related by the equation ————

6. Let $A(x)$ be the area under the graph of a nonnegative continuous function f over an interval $[a, x]$, then $A'(x)=$ ————.

7. Integrals over infinite intervals are known as ————

P.T.O.

8. $\cosh x + \sinh x =$ _____.

9. Define the work done by a force F.

10. The total mass of a homogeneous lamina of area A and density δ is _____.

(10 × 1 = 10 Marks)

SECTION – II

Answer **any eight** questions. Each question carries **2** marks.

11. Evaluate $\lim_{x \to 0^+} \left(\frac{1}{x} - \frac{1}{\sin x} \right)$.

12. Find the subintervals of $[0, 2\pi]$ in which the function $f(x) = x + 2\sin x$ is decreasing.

13. Find all critical points of $f(x) = x^3 - 3x + 1$.

14. What are the geometrical implications of the multiplicity of a root of a polynomial?

15. Find the horizontal and vertical asymptotes of the curve given by $y = \frac{\ln x}{x}$.

16. Find the absolute extrema of $f(x) = 6x^{4/3} - 3x^{1/3}$ on the interval $[-1, 1]$.

17. Suppose that a particle moves on a coordinate line so that its velocity at time t is $v(t) = t^2 - 2t \; m/s$. Find the distance traveled by the particle during the time interval $0 \le t \le 3$.

18. Find the average value of the function $f(x) = \sqrt{x}$ over the interval [1, 4].

19. Define hyperbolic sine and draw its graph.

20. Define improper integral. Is $\int_0^3 \frac{dx}{x^2 - 3x + 2}$ an improper integral? Explain.

21. Use Pappus Theorem to find the volume V of the torus generated by revolving a circular region of radius *b* about a line at a distance a (greater than *b*) from the rater of the circle.

22. Evaluate $\int_0^{\infty} e^{-x} dx$.

(8 × 2 = 16 Marks)

SECTION – III

Answer **any six** questions. Each question carries **4** marks.

23. Evaluate $\lim_{x \to 0} (\cos x)^{1/x^2}$.

24. Find all the inflection points of $f(x) = xe^{-x}$.

25. Find the radius and height of the right circular cylinder of largest volume that can be inscribed in a right circular cone with, radius 6 inches and height 10 inches.

26. State and prove Rolle's theorem.

27. Find the volume of the solid generated when the region between the graphs of the equations $f(x) = \frac{1}{2} + x^2$ and $g(x) = x$ over the interval [0, 2] is revolved about the *x*-axis.

28. Using the notion of surface of revolution, show that the area of the surface of a sphere of radius r is $4\pi r^2$.

29. Find the length of the arc of the curve $y^2 = x^3 3$ from the origin to the point (1, 1).

30. A spring exerts a force of 5 N when stretched 1 *m* beyond its natural length.

    (a) Find the spring constant *k*.

    (b) How much work is required to stretch the spring 1.8 m beyond its natural length?

31. Evaluate $\int_0^{\infty} (1 - x) e^{-x} dx$.

(6 × 4 = 24 Marks)

SECTION – IV

Answer **any two** questions. Each question carries **15** marks.

32. (a) Find the radius and height of the right circular cylinder of largest volume that can be inscribed in a right circular cone with radius 6 inches and height 10 inches. **5**

(b) Using Roll's theorem show that between any two real root of $e^{-x} = \sin x$, there is at least one real root of $e^{-x} = -\cos x$. **5**

(c) Find the points of inflection of the cubic $y = \dfrac{a^2 x}{x^2 + a^2}$. **5**

33. (a) Explain the 7 steps in sketching the graph of a rational function. **6**

(b) Sketch the graph of $y = \dfrac{x^2 - 1}{x^3}$. **9**

34. (a) Find the length of the curve $y = \log \sec x$ between the points given by $x = 0$ and $x = \pi/3$. **5**

(b) Find the volume when the loop of the curve $y^2 = x(2x-1)^2$ revolves about the x-axis. **5**

(c) Find the area of the surface that is generated by revolving the portion of the curve $y = x^2$ between $x = 1$ and $x = 2$ about the y-axis. **5**

35. (a) A space probe of mass $m = 5.00 \times 104$ kg travels in deep space subjected only to the force its own engine. Starting at a time when the speed of the probe is v = 1.10 x 104 m/s. the engine is fired continuously over a distance of 2.50 × 106 m with a constant force of 400 × 105 N in the direction of motion. What is the final speed of the probe? **6**

(b) Evaluate $\displaystyle\int_1^4 \frac{dx}{(x-2)^{2/3}}$. **5**

(c) Find the mass and center of gravity of the lamina bounded by the x-axis, the line x = 1, and the curve $y = \sqrt{x}$. Given $\delta = 2$. **4**

(2 × 15 = 30 Marks)
