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Reg. No. : ....

Name : ....

First Semester B.Sc. Degree Examination, June 2022

First Degree Programme under CBCSS

Mathematics

Complementary Course I for Chemistry and Polymer Chemistry

MM 1131.2 : MATHEMATICS I – CALCULUS WITH APPLICATIONS IN CHEMISTRY – I

(2020 Admission)

Time : 3 Hours Max. Marks : 80

SECTION – I

All the **first ten** questions are **compulsory**. They carry **1** mark each.

1. Find the first derivative of $\cos 2x$.
2. Find the 1000^{th} derivative of e^x.
3. Define stationary point.
4. State Leibnitz's Theorem.
5. State Demoivre's Theorem.
6. Define argument of a complex number.
7. Find the complex conjugate of $2-2i$.
8. If $v = 3i - 4j$ is a velocity vector. Then find speed.
9. Define dot product.
10. Evaluate $\int x \sin x$.

(10 × 1 = 10 Marks)

P.T.O.

SECTION – II

Answer any **eight** questions. These question carries **2** marks each.

11. If $x = \sec t$ and $y = \tan t$, find $\dfrac{dy}{dx}$.

12. Find $\dfrac{dy}{dx}$, if $e^x - \sin y = x$.

13. Express $\dfrac{(x - iy)^2}{(x + iy)}$ in the form $a + bi$.

14. Explain multiplication and division of complex numbers in polar form.

15. Find modulus of $6 + 8i$.

16. Find the value of $Ln(-3i)$.

17. Find $\dfrac{d}{dx}(\cosh x)$.

18. Find the angle between two vectors a and b with magnitudes $\sqrt{3}$ and 2 respectively, and such that $a \cdot b = \sqrt{6}$.

19. Find the value of p for which the vectors $3i + 2j + 9k$ and $i + pk + 3k$ are perpendicular.

20. Show that if $a = b + \lambda c$, for some scalar λ, then $a \times c = b \times c$.

21. Two particles have velocities $v_i = i + 3j + 6k$ and $v_2 = i + 2k$, respectively. Find the velocity u of the second particle relative to the first.

22. Find the unit vector corresponding to the vector $i + j + k$.

23. Find the area of the parallelogram with sides $i + 2j + 3k$ and $4i + 5j + 6k$.

24. Evaluate $\displaystyle\int_0^2 \frac{1}{(2 - x)^{\frac{1}{4}}} dx$.

25. Evaluate $\displaystyle\int_0^\infty \frac{x}{(x^2 + a^2)^2} dx$.

26. Find the mean value in of the function $f(x) = 3x^2 - 3$ between the $x = 0$ and $x = 1$.

(8 $\times$ 2 = 16 Marks)

SECTION – III

Answer any **six** questions. These question carries **4** marks each.

27. Find the second derivative of $\frac{x^4+5x^3}{2x^2+1}$.

28. Find the magnitude of radius of curvature at a point (x, y) on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.

29. Find the positions and nature of points of the function $f(x)=3x^4-4x^3-8$.

30. Prove that $\cosh^2 x-\sinh^2 x=1$.

31. Solve the hyperbolic equation $\cosh x-5\sinh x-5=0$.

32. Find the value of $(1+i)^i$.

33. Find an expression for $\cos^3\theta$ in terms of $\cos 3\theta$ and $\cos\theta$.

34. Evaluate $\int e^{ax}\cos bx\,dx$.

35. Find the volume of a parallelepiped with sides $2i-4j+5k$, $i-j+k$ and $3i-5j+2k$.

36. The vertices of triangle *ABC* have position vectors *a, b* and *c* relative to some origin *O*. Find the position vector of the centroid *G* of the triangle.

37. A line is given by $r=a+\lambda b$, where $a=i+2j+3k$ and $b=4i+5j+6k$. Find the coordinates of the point *P* at which the line intersects the plane $x+2y+3z=6$.

38. Find the volume of a cone enclosed by the surface formed by rotating the curve $y=2x$ about the x – axis the line between $x=0$ and $x=h$.

(6 $\times$ 4 = 24 Marks)

SECTION – IV

Answer any **two** questions. These question carries **15** marks each.

39. (a) Determine inequalities satisfied by *In* x *and sin x* for suitable ranges of the real variable *x*.

 (b) Determine the constants *a* and b so that the curve $y=x^3+ax^2+bx$ has a stationary point inflection at the point (3, –9).

40. (a) State Rolle's Theorem.

(b) What semi-quantitative results can be deduced by applying Rolle's theorem to the following functions *f(x),* with a and c chosen so that $f(a) = f(x) = 0$?

(i) $\sin x$

(ii) $x^2 - 3x + 2$

(iii) $2x^3 - 9x^2 - 24x + k$.

41. (a) Express $\cosh^{-1} x$ in logarithmic form.

(b) Evaluate $\frac{d}{dx}\left(\sin^{-1} x\right)$.

42. (a) A point *P* divides a line segment *AB* in the ratio $\lambda : \mu$. If the position vectors of the points *A* and *B* are *a* and *b* respectively, find the position vector of the point *P*.

(b) Find the minimum distance from the point *P* with coordinates (1, 2, 1) to the line $r = a + \lambda b$, where $a = i + j + k$ and $b = 2i - j + 3k$.

43. (a) Using integration by parts, find a relationship between I_n and I_{n-1} where $I_n = \int_0^1 \left(1 - x^3\right)^n dx$ and n is any positive integer. Hence evaluate $I_2 = \int_0^1 \left(1 - x^3\right)^2 dx$.

(b) Find the surface area of a cone formed by rotating about the *x*-axis the line $y = 2x$ between $x = 0$ and $x = h$.

44. (a) The equation in polar coordinates of an ellipse with semi-axes *a* and *b* is $\frac{1}{p^2} = \frac{\cos^2 \phi}{a^2} + \frac{\sin^2 \phi}{b^2}$. Find the area of the ellipse.

(b) Find the length of the curve $y = x^{3/2}$ from $x = 0$ to $x = 2$.

(2 × 15 = 30 Marks)