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Reg. No. : ....

Name : ....

Third Semester B.Sc. Degree Examination, January 2023

First Degree Programme under CBCSS

Mathematics

Complementary Course for Physics

MM 1331.1 — MATHEMATICS III – CALCULUS AND LINEAR ALGEBRA

(2019 – 2020 Admission)

Time : 3 Hours

Max. Marks : 80

SECTION – I

All the **ten** questions are compulsory. They carry **1** mark each.

1. Find the order of the ODE $\frac{d^3y}{dx^3}+x\left(\frac{dy}{dx}\right)^{\frac{3}{2}}+x^2y=0$.

2. Check whether the equation $(3x+y)dx+xdy=0$ is exact or not.

3. Write the general form of Euler's linear equation.

4. Prove that $\nabla\times\bar{r}=0$.

5. State Stoke's theorem.

6. Find the average value of the function $f(x)=1-e^{-x}$ on (0, 1).

7. What are the Fourier coefficients of an odd function $f(x)$ in the interval $(-l,l)$.

P.T.O.

8. Find the matrix product $\begin{pmatrix} 2 & 3 \end{pmatrix}\begin{pmatrix} -1 & 4 \\ 2 & -1 \end{pmatrix}\begin{pmatrix} -1 \\ 2 \end{pmatrix}$.

9. Define Hermitian matrix.

10. Define trace of a matrix.

(10 × 1 = 10 Marks)

SECTION – II

Answer any **eight** questions. These questions carry **2** marks each.

11. Solve : $y' - xy^3 = 0$.

12. Solve : $x\dfrac{dy}{dx} + 3x + y = 0$.

13. Find a particular integral of the equation $\dfrac{d^2y}{dx^2} - 2\dfrac{dy}{dx} + y = e^x$.

14. Solve : $\dfrac{dy}{dx} + 2xy = 4x$.

15. Solve : $x^2\dfrac{d^2y}{dx^2} - x\dfrac{dy}{dx} + y = 0$.

16. Calculate $\int_C \vec{F}\cdot d\vec{r}$ where $\vec{F} = y^2\,\vec{i} - x^2\,\vec{j}$ along the parabola $y = 4x^2$ from (0, 0) to (1, 4).

17. Find an expression for the angular momentum of a solid body rotating with angular velocity w about an axis through the orgin.

18. Check whether $\vec{F} = e^y \sin x\,\vec{i} + e^y \cos x\,\vec{j} - 2z e^y \cos x\,\vec{k}$ is solenoidal.

19. Write the complex form of Fourier series.

20. What are the Dirichlet conditions for the existence of the Fourier series of a periodic function?

21. Write Fourier cosine Transforms.

22. Find the row reduced matrix of $\begin{pmatrix} 2 & 0 & -1 & 2 \\ 6 & 5 & 3 & 7 \\ 2 & -1 & 0 & 4 \end{pmatrix}$.

23. Show without computation that the following determinant is equal to zero, $\begin{vmatrix} 0 & 2 & -2 \\ -2 & 0 & 4 \\ 2 & -4 & 0 \end{vmatrix}$.

24. Find the angle between the lines $2x + 6y - 3z = 0$ and $5x + 2y - z = 12$.

25. Show that the functions $1, x, \sin x$ are linearly independent.

26. Define symmetric matrix and prove that AA^T is a symmetric matrix for any matrix A.

(8 × 2 = 16 Marks)

SECTION – III

Answer any **six** questions. These questions carry **4** marks each.

27. Solve : $\frac{dy}{dx} = -\frac{2}{y} - \frac{3y}{2x}$.

28. Solve : $\frac{dy}{dx} = \frac{y}{x} + \tan\left(\frac{y}{x}\right)$.

29. Solve : $\frac{dy}{dx} = \frac{2x - 5y + 3}{2x + 4y - 6}$.

30. Solve : $xp^2 + 2xp - y = 0$.

31. Evaluate the line integral $I = \oint_C x\,dy$, where C is the circle in the xy plane defined by $x^2 + y^2 = a^2, z = 0$.

32. A vector field $\bar{a} = f(r)\bar{r}$ is spherically symmetric and every where directed away from the origin. Show that $\bar{a}$ is solenoidal of $f(r)$ is of the form $A\bar{r}^{3}$.

33. Given $f(x) = \begin{cases} 0 & 0 < x < l \\ 1 & l < x < 2l \end{cases}$. Expand $f(x)$ is an exponential fourier series of period $2l$.

34. Prove that fourier expansion of $f(x) = \begin{cases} 0 & -\pi < x < 0 \\ 1 & 0 < x < \pi \end{cases}$ is $f(x) = \frac{1}{2} + \frac{2}{\pi}\left(\frac{\sin x}{1} + \frac{\sin 3x}{3} + \frac{\sin 5x}{5} + \ldots\right)$.

35. Find the rank of the matrix $\begin{pmatrix} 1 & 1 & 4 & 3 \\ 3 & 1 & 10 & 7 \\ 4 & 2 & 14 & 10 \\ 2 & 0 & 6 & 4 \end{pmatrix}$.

36. Find the distance between the lines $\vec{r} = \vec{i} - 2\vec{j} + (\vec{i} - \vec{k})t$ and $\vec{r} = 2\vec{j} + \vec{k} + (\vec{j} - \vec{i})t$.

37. Find the inverse of the matrix $\begin{pmatrix} -1 & 2 & 3 \\ 2 & 0 & -4 \\ -1 & -1 & 1 \end{pmatrix}$.

38. Solve the set of homogenous equations $2x + 3z = 0$; $4x + 2y + 5z = 0$; $x - y + 2z = 0$ by row reducing the matrix.

(6 × 4 = 24 Marks)

SECTION – IV

Answer any **two** questions. These questions carry **15** marks each.

39. (a) Solve : $\frac{d^2y}{dx^2} + 4y = x^2 \sin 2x$.

(b) Solve : $x^3 \frac{d^2y}{dx^2} - \left(x^2 + xy\right)\frac{dy}{dx} + \left(y^2 + xy\right) = 0$.

40. Express the equation $\frac{d^2y}{dx^2} + 4x\frac{dy}{dx} + \left(4x^2 + 6\right)y = e^{-x^2} \sin 2x$ in canonical form and hence find the general solution.

41. The vector field $\vec{F}$ given by $\vec{F} = \left(3x^2yz + y^3z + xe^{-x}\right)\vec{i} + \left(3xy^2z + x^3z + ye^{x}\right)\vec{j} + \left(x^3y + y^3x + xy^2z^2\right)\vec{k}$. Calculate

(a) directly, and

(b) by using Stoke's theorem the value of the line integral $\int_L \vec{F} \cdot d\vec{r}$, where L is the (three dimensional) closed contour OABCDEO defined by the successive vertices (0, 0, 0), (1, 0, 0), (1, 0, 1), (1, 1, 1), (1, 1, 0), (0, 1, 0), (0, 0, 0).

42. Let $f(x) = x^2, -\pi < x < \pi$. Find

(a) a Fourier sine series

(b) a Fourier cosine series

(c) a complex exponential Fourier series.

43. (a) Find the distance from the point $P(1,-2,3)$ to the plane $3x-2y+z+1=0$.

(b) Use Cramer's rule to solve the set of equations $2x-z=2$, $6x+5y+3z=7$; $2x-y=4$.

(c) Show that $\begin{vmatrix} \cos\theta & 1 & 0 \\ 1 & 2\cos\theta & 1 \\ 0 & 1 & 2\cos\theta \end{vmatrix} = \cos 3\theta$.

44. (a) Find the eigen values and eigen vectors of the matrix $\begin{pmatrix} 2 & 0 & 2 \\ 0 & 2 & 0 \\ 0 & 2 & -1 \end{pmatrix}$.

(b) Show that the product of two unitary matrices is unitary.

(2 × 15 = 30 Marks)
