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Reg. No. : ....

Name : ....

Third Semester B.Sc. Degree Examination, January 2023

First Degree Programme under CBCSS

Mathematics

Core Course

MM 1341 : ELEMENTARY NUMBER THEORY AND CALCULUS – I

(2018 Admission)

Time : 3 Hours

Max. Marks : 80

SECTION – A

Answer **all** questions.

1. For every positive integer *n*, find *n* consecutive integers that are composite numbers.

2. Prove that there are infinitely many primes.

3. State Dirichlet's theorem.

4. State the Pigeonhole principle.

5. If $r(t) = t^2 i + e^t j - (2\cos \pi t)k$, find $r'(t)$.

6. Prove that a straight line has zero curvature at every point.

7. Evaluate: $\int_0^2 r(t)dt$, where $r(t) = 2ti + 3t^2 j$.

P.T.O.

8. If f is a function of x, y and z, what is the gradient of f?

9. State the chain rule for partial derivatives if $z = f(x, y)$, $x = x(u)$, $y = y(u)$.

10. Let $f(x, y) = y^2 e^x + y$. Evaluate f_{xyy}.

(10 × 1 = 10 Marks)

SECTION – B

Answer any **eight** questions.

11. Using recursion, evaluate, (18, 30, 60, 75, 132).

12. Derive a necessary and sufficient condition for two positive integers to be relatively prime.

13. Prove that (a, b) = (a, a – b).

14. Find the number of positive integers ≤ 2076 and divisible by neither 4 nor 5.

15. If $r(t)$ is a differentiable vector - valued function in 2 – space or 3-space and $\|r(t)\|$ is constant for all t, then show that $r(t)$ and $r'(t)$ are orthogonal vectors for all t.

16. State the Newton's laws of universal gravitation.

17. Show that the circle of radius a which centred at the origin has constant curvature $\frac{1}{a}$.

18. Evaluate the unit tangent vector to the graph of $r(t) = t^2 i + t^3 j$ *at* the point where $t = 2$.

19. Estimate an equation for the tangent plane and parametric equations for the normal line to the surface $z = x^2 y$ at the point (2, 1, 4).

20. Find the directional derivative of $f(x, y, z) = x^2 y - yz^3 + z$ at (1, –2, 0) in the direction of the vector $a = 2i + j - 2k$.

21. Evaluate $f_x(1, 3)$ and $f_y(1, 3)$ by finding $f_x(x, y)$ and $f_y(x, y)$ where $f(x,y) = 2x^3y^2 + 2y + 4x$.

22. Prove that $f(x, y) = x^2 + y^2$ is differentiable at the origin.

(8 × 2 = 16 Marks)

SECTION – C

Answer any **six** questions.

23. Let e denote the highest power of 2 that divides $n!$ and b the number of 1s in the binary representation of n. Then show that $n = e + b$.

24. Show that the gcd of the positive integers a and b is a linear combination of a and b.

25. Show that 3, 5 and 7 are the only three consecutive odd integers that are primes.

26. Find parametric equations of the tangent line to the circular helix $x = \cos t$, $y = \sin t$, $z = t$ where $t = t_0$, and use that result to find parametric equations for the tangent line at the point $t = \pi$.

27. Find the curvature of the ellipse with vector equation $r = 2\cos t\, i + 3\sin t\, j$, $(0 \le t \le 2\pi)$ at the end points of the major and minor axes.

28. Derive Kepler's third law.

29. Find the slope of the sphere $x^2 + y^2 + z^2 = 1$ in the y – direction at the points $\left(\frac{2}{3}, \frac{1}{3}, \frac{2}{3}\right)$ and $\left(\frac{2}{3}, \frac{1}{3}, -\frac{2}{3}\right)$.

30. Verify whether the function $z = e^x \sin y + e^y \cos x$ satisfies the equation $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = 0$.

31. Locate all relative extrema and saddle points of $f(x, y) = 3x^2 - 2xy + y^2 - 8y$.

(6 × 4 = 24 Marks)

SECTION – D

Answer any **two** questions.

32. (a) Prove that there is no polynomial $f(n)$ with integral coefficients that will produce primes for all integers n.

    (b) Find the general solution of the LDE $6x + 8y + 12z = 10$.

33. (a) State and prove the division algorithm.

    (b) Show that the number of leap years l after 1600 and not exceeding a given year y is given by $l = [y/4] - [y/100] + [y/400] - 388$.

34. Suppose that a particle moves through 3 – space so that its position vector at time t is $r(t) = ti + t^2 j + t^3 k$.

    (a) Find the scalar tangential and normal components of acceleration at time t.

    (b) Find the scalar tangential and normal components of acceleration at time $t = 1$.

    (c) Find the vector tangential and normal components of acceleration at time $t = 1$.

    (d) Find the curvature of the path at the point where the particle is located at time $t = 1$.

35. Find the points on the sphere $x^2 + y^2 + z^2 = 36$ that are closest to and farthest from the point (1, 2, 2).

(2 × 15 = 30 Marks)