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R – 3116

Reg. No. : ....

Name : ....

Second Semester B.Sc. Degree Examination, September 2023

First Degree Programme under CBCSS

Mathematics

Complementary Course for Physics

MM 1231.1 : MATHEMATICS II – APPLICATIONS OF CALCULUS AND VECTOR DIFFERENTIATION

(2021 Admission Onwards)

Time : 3 Hours

Max. Marks : 80

SECTION – A

All the first **Ten** questions are compulsory. They carry **1** mark each.

1. Show that the function $f(x) = x^3 + 6x^2 + 12x - 9$ is increasing for all values of x.

2. Define inflection point of a function.

3. Determine where the graph of the function $f(x) = x + 2\sin x$ is concave up.

4. Define critical point of a function.

5. State Rolle's theorem.

6. Evaluate $\int_0^1 \int_0^2 (x + 5)\, dy\, dx$.

7. Does L'Hopital's rule apply to the limit, $\lim_{x \to 0} \left(\frac{\cos x}{x} \right)$?

P.T.O.

8. Find the rectangular co-ordinates of the point whose polar co-ordinates are given by $(r,\theta) = (4, \pi/6)$.

9. Identify the quadric surface $z = \frac{x^2}{4} + \frac{y^2}{25}$.

10. Find $r'(t)$ if $r(t) = \cos t\, i + \sin t\, j + t\, k$.

(10 × 1 = 10 Marks)

SECTION – B

Answer any **eight** questions. These questions carry **2** marks each.

11. Find the x–coordinates of all inflection points of $x^4 - 5x^3 + 9x^2$.

12. Let $f(x) = x^2 + px + q$. Find the values of p and q such that f (1) = 3 is an extreme value of f on [0, 2]. Is this value a maximum or minimum?

13. Find the length of the curve y = 2x from (1, 2) and (2, 4).

14. Find the total area between the curve $y = 1 - x^2$ and the x axis over the interval [0, 2].

15. Find area of the surface that is generated by revolving the portion of the curve $y^2 = x$ from origin to the point where x = 2 about the x–axis.

16. Evaluate : $\int_{-1}^{2}\int_{0}^{2}\int_{0}^{1}(x^2 + y^2 + z^2)\, dxdydz$.

17. Find the length of the arc of the parabola $y^2 = 4ax$ from the vertex to the extremity of the latus rectum.

18. Find the area of the surface that is generated by revolving the curve $y = \sqrt{4 - x^2}, -1 \le x \le 1$.

19. Prove that the curve $r(t) = \left(2t, \frac{2}{1+t^2}\right)$ is also the graph of the function $y = \frac{8}{4+x^2}$.

20. Describe the graph of $r(t) = (1+2t, -1+3t)$.

21. If f(x,y) = xy, find D_u f (1, 2) for the unit vector $u = \frac{\sqrt{3}}{2}i + \frac{1}{2}j$.

22. Find the natural domain of the vector valued function $r(t) = \cos \pi t \; i - \ln t \; j + \sqrt{t-4} \; k$.

(8 × 2 = 16 Marks)

SECTION – C

Answer any **six** questions. These questions carry **4** marks each.

23. Verify Rolle's theorem for the function $f(x) = x^3 - 9x$.

24. Evaluate $\lim_{x \to 0} \left(\frac{1}{x} - \frac{1}{\sin x} \right)$.

25. Find by double integration, the area lying between the parabola $y = 4x - x^2$ and the line $y = x$.

26. Find the length of the curve $27y^2 = 4x^3$ from (0,0) to (3,2).

27. Find the entire length of the astroid $x^{2/3} + y^{2/3} = a^{2/3}$.

28. Use the Maclaurin series for $\frac{1}{1+x^2}$ to find the Maclaurin series for $\tan^{-1} x$.

29. Evaluate the integral $\int_0^2 \int_0^{\sqrt{4-x^2}} (x^2 + y^2) \, dydx$ by converting to polar coordinates.

30. If $r_1(t) = 5t\,i + t\,j - t^4\,k$ and $r_2(t) = \sin t\,i - \cos t\,j$. Find $\frac{d}{dt}(r_1 \cdot r_2)$ and $\frac{d}{dt}(r_1 \times r_2)$.

31. Find the directional derivative of $f(x,y,z) = x^2y - yz^3 + z$ at the point (1,–2,0) in the direction of the vector $a = 2i + j - 2k$.

(6 × 4 = 24 Marks)

SECTION – D

Answer any **two** questions. These question carry **15** marks each.

32. (a) Use cylindrical shells to find the volume of the solid generated when the region enclosed between $y = x^3 - 3x^2 + 2x$ over [0, 1] is revolved about the y–axis.

(b) Find the area enclosed by the curve $x^{2/3} + y^{2/3} = a^{2/3}$.

33. (a) Find the area of the surface that is generated by revolving the portion of the curve $y = x^3$ between x = 0 and x = 1 about the x–axis.

(b) Find the volume of the solid generated when the region between the graphs of the equations $f(x) = \frac{1}{2} + x^2$ and g(x) = x over the interval [0, 2] is revolved about the x–axis.

34. (a) Use the relationship $\int \frac{1}{\sqrt{1-x^2}}\,dx = \sin^{-1} x + C$ to find the first four nonzero terms in the Maclaurin series for $\sin^{-1} x$.

(b) Evaluate $\lim_{x \to 0} \frac{x - \sin x}{x^3}$ in two ways; using L'Hopital's rule and by replacing sin x by its Maclaurin's series.

35. (a) Use triple integration to find the volume of the tetrahedron in the first octant bounded by the coordinate surfaces x = 0, y = 0, and z = 0 and the plane $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$ (a>0, b>0, c>0).

(b) Use triple integration in cylindrical coordinates to evaluate $\int_{-3}^{3} \int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}} \int_{0}^{9-x^2-y^2} x^2 \, dzdydx$.

(2 × 15 = 30 Marks)
