(Pages : 4) **R – 3115**

Reg. No. : ....

Name : ....

Second Semester B.Sc. Degree Examination, September 2023

First Degree Programme under CBCSS

Mathematics

Complementary Course for Physics

MM 1231.1 : MATHEMATICS II – CALCULUS WITH APPLICATIONS IN PHYSICS – II

(2018 – 2020 Admission)

Time : 3 Hours Max. Marks : 80

SECTION I

All 10 questions are compulsory. Each question carries 1 mark.

1. Represent $1+2i$ in Argand diagram.
2. Find $(3+4i)+(18+10i)$.
3. State De-Moirre's theorem.
4. Find $\frac{\partial f}{\partial x}$ if $f(x,y)=2x+y$.
5. Write Taylor series expansion for functions of 2 variables.
6. Evaluate $\int_0^2\int_{-1}^1 (x-y)\,dy\,dx$.
7. Write down the formula for finding the area of a closed bounded region R.
8. State Pappus theorem.

P.T.O.

9. Find velocity of a particle if $r(t) = (t+1)\bar{i} + (t^2 - 1)\bar{j} + 2t\bar{k}$.

10. Find the divergence of the vector field $a = x^2y^2\bar{i} + y^2z^2\bar{j} + x^2z^2\bar{k}$.

(10 × 1 = 10 Marks)

SECTION II

Answer **any eight** questions. Each question carries **2** marks.

11. Express z in the form of $x + iy$ where $z = \dfrac{3-2i}{-1+4i}$.

12. Prove that $\cos 4\theta = \cos^4\theta - 6\cos^2\theta\sin^2\theta + \sin^4\theta$.

13. Prove that $\dfrac{d}{dx}(\cosh x) = \sinh x$.

14. Verify that $|z_1 z_2| = |z_1||z_2|$, where $z_1 = 3 + 2i$ and $z_2 = -1 - 4i$.

15. Find the total differential of the function $f(x,y) = ye^{x+y}$.

16. Check whether the differential equation $(y^2 - 2x)dx + (2xy + 1)dy = 0$ is exact or not.

17. Given that $x(u) = 1 + au$ and $y(u) = bu^3$, find the rate of change of $f(x,y) = xe^{-y}$ with respect to u.

18. Define the partial derivative of $f(x,y)$ with respect to y.

19. Calculate $\iint_R f(x,y)\,dx\,dy$ if $f(x,y) = 100 - 6x^2y$ and $R: 0 \le x \le 2$ and $-1 \le y \le 1$.

20. Write any two properties of 'Jacobian'.

21. Find the curl of the vector field $x^2z\bar{i} - 2xz\bar{j} + yz\bar{k}$.

22. Find the Laplacian of the scalar field $\phi = xy^2z^3$.

(8 × 2 = 16 Marks)

SECTION III

Answer **any six** questions. Each question carries **4** marks.

23. Express $\sin 5\theta$ in terms of $\sin\theta$.

24. Prove that $\cosh^{-1} x = \log\left[x + \sqrt{x^2 - 1}\right]$.

25. Find the solution to the equation $z^3 = 1$.

26. Find the Taylor series expansion upto quadratic terms in $(x-2)$ and $(y-3)$ of $f(x,y) = ye^{xy}$ about the point $x = 2$ and $y = 3$.

27. Find all the second order partial derivatives of the function $f(x,y) = 2x^2y + x^2y^6 - 1$.

28. Find the centre of mass of the solid hemisphere bounded by the surface $x^2 + y^2 + z^2 = a^2$ and the XY plane assuming that it has a uniform density?

29. Find the volume of the region bounded by the paraboloid $z = x^2 + y^2$ and the plane $z = 2y$.

30. A semicircular uniform lamina is freely suspended from one of its corners. Show that its straight edge makes an angle of 23.0° with the vertical.

31. For the function $\phi = x^2y + yz$ at the point $(1, 2, -1)$, find its rate of change with distance in the direction $a = \bar{i} + 2\bar{j} + 3\bar{k}$. At the same point, what is the greatest possible rate of change with distance and in which direction does it occur.

(6 × 4 = 24 Marks)

SECTION IV

Answer **any two** questions. Each question carries **15** marks.

32. (a) By Integrating $e^{(a+ib)x}$, find $\int e^{ax} \cos bx\, dx$ and $\int e^{ax} \sin bx\, dx$.

(b) Prove that $\sin^6\theta = -\frac{1}{2^5}[\cos 6\theta - 6\cos 4\theta + 15\cos 2\theta - 10]$.

33. (a) Solve the equation $z^6 - z^5 + 4z^4 - 6z^3 + 2z^2 - 8z + 8 = 0$.

(b) Plane polar co-ordinates, ρ and ϕ and the Cartesian co-ordinates x and y are related by the expressions

$x = \rho\cos\phi$ and $y = \rho\sin\phi$. An arbitrary function $f(x,y)$ can be re-expressed as a function $g(\rho,\phi)$. Transform the expression $\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$ into one in ρ and ϕ.

34. (a) Find the mass of the tetrahedron bounded by the 3 co-ordinate surfaces and the plane $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$, if the density is given by $\rho(x,y,z) = \rho_0\left(1 + \frac{x}{a}\right)$.

(b) Find the area of the region enclosed by the parabola $y = x^2$ and the line $y = x + 2$

35. (a) Express the vector field $a = yz\bar{i} - y\bar{j} + xz^2\bar{k}$ in cylindrical polar co-ordinates and hence calculate its divergence.

(b) Show that the acceleration of a particle travelling along a trajectory $r(t)$ is given by $a(t) = \frac{dv}{dt}\hat{t} + \frac{v^2}{\rho}\hat{n}$, where v is the speed of the particle, $\hat{t}$ is the unit tangent to the trajectory, $\hat{n}$ is its principal normal and ρ is its radius of curvature.

(2 × 15 = 30 Marks)