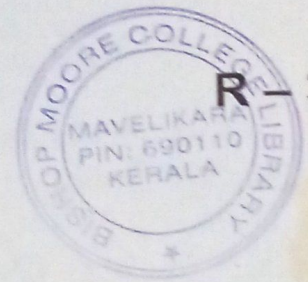


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R-3132

Reg. No. : .....

Name : .....

Second Semester B.Sc. Degree Examination, September 2023

First Degree Programme under CBCSS

Mathematics

Complementary Course for Chemistry / Polymer chemistry

MM 1231.2 : MATHEMATICS II – INTEGRAL CALCULUS AND  
VECTOR DIFFERENTIATION

(2021 Admission Onwards)

Time : 3 Hours

Max. Marks : 80

SECTION – A

Answer all questions.

1. Give an example of a proper rational function.
2. Evaluate:  $\int \sin 7x \cos 3x \, dx$ .
3. Write the formula for integration by parts.
4. What is a surface of revolution?
5. Write the second – order model of the period T of a simple pendulum.
6. Find the parametric equations for the surface generated by revolving the curve  $y = \frac{1}{x}$  about the x – axis.

7. Write the conversion formulas for spherical coordinate system to rectangular coordinate system.
8. If  $r(t) = t^2i + e^tj - (2\cos \pi t)k$ , find  $r'(t)$ .
9. Define the gradient of  $f(x, y, z)$ .
10. Evaluate  $\int_0^2 (2ti + 3t^2j) dt$ .

(10 × 1 = 10 Marks)

### SECTION – B

Answer any **eight** questions.

11. Evaluate :  $\int \tan^2 x \sec x dx$ .
12. Solve the initial value problem  $\frac{dy}{dx} = \cos x$ ,  $y(0) = 1$ .
13. Using integration by parts, compute :  $\int x^2 \sqrt{x-1} dx$ .
14. Estimate :  $\int \frac{e^{\tan^{-1}x}}{1+x^2} dx$ .
15. Compute :  $\int_0^2 x(x^2 + 1)^3 dx$
16. Find the volume of the solid generated when the region enclosed by  $y = \sqrt{x}$ ,  $y = 2$  and  $x = 0$  is revolved about the  $y$  – axis.
17. Find the area of the region enclosed by  $x = y^2$  and  $y = x - 2$ .
18. Use polar coordinates to compute  $\int_{-1}^1 \int_0^{\sqrt{1-x^2}} (x^2 + y^2)^{3/2} dy dx$ .

19. Find the partial derivatives of  $r = ui + vj + (4 - u^2 - v^2)k$ .
20. Find the polar coordinates of the point whose rectangular coordinates are  $(-2, -2\sqrt{3})$ .
21. Let  $f(x, y) = xy$ . Find  $D_u f(1, 2)$  for the unit vector  $u = \frac{\sqrt{3}}{2}i + \frac{1}{2}j$ .
22. Find the directional derivative of  $f(x, y, z) = x^2y - yz^3 + z$  at the point  $(1, -2, 0)$  in the direction of the vector  $a = 2i + j - 2k$ .

(8 × 2 = 16 Marks)

### SECTION – C

Answer any **six** questions.

23. Separate  $\frac{2x + 4}{x^3 - 2x^2}$  into partial fractions.
24. Evaluate :  $\int_0^1 \tan^{-1} x \, dx$ .
25. Find the total area between the curve  $y = 1 - x^2$  and the  $x$  - axis over the interval  $[0, 2]$ .
26. Find the volume of the solid generated when the region between the graphs of the equations  $f(x) = \frac{1}{2} + x^2$  and  $g(x) = x$  over the interval  $[0, 2]$  is revolved about the  $x$  - axis.
27. Prove :  $\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots$
28. Estimate  $\iint_A \sin \theta \, dA$  where  $R$  is the region in the first quadrant that is outside the circle  $r = 2$  and inside the cardioid  $r = 2(1 + \cos \theta)$ .

29. Find  $\iint_R (2x - y^2) dA$  over the triangular region  $R$  enclosed between the lines  $y = -x + 1$ ,  $y = x + 1$  and  $y = 3$ .
30. Compute the volume of the solid that is bounded above by the plane  $z = 4 - x - y$  and below by the rectangle  $R = [0, 1] \times [0, 2]$ .
31. Evaluate  $\int_1^3 \int_2^4 (40 - 2xy) dy dx$ .

(6 × 4 = 24 Marks)

SECTION – D

Answer any **two** questions.

32. (a) Find the area of the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ .
- (b) Evaluate :  $\int e^x \cos x dx$
33. (a) Calculate the area of the surface that is generated by revolving the portion of the curve  $y = x^2$  between  $x = 1$  and  $x = 2$  about the  $y$ -axis.
- (b) Use cylindrical shells to find the volume of the solid generated when the region under  $y = x^2$  over the interval  $[0, 2]$  is revolved about the line  $y = -1$ .
34. Sketch the graph  $r = a(1 - \cos \theta)$  in polar coordinates, assuming  $a$  to be a positive constant.
35. (a) A heat – seeking, particle is located at the point  $(2, 3)$  on a flat metal plate whose temperature at  $(x, y)$  is  $T(x, y) = 10 - 8x^2 - 2y^2$ . Find an equation for the trajectory of the particle if it moves continuously in the direction of maximum temperature increase.
- (b) Evaluate :  $\int_0^1 \int_{-x}^{x^2} y^2 x dy dx$ .
- (c) Estimate :  $\int_0^{\pi/3} \int_0^{\cos y} x \sin y dx dy$ .

(2 × 15 = 30 Marks)