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Reg. No. : .....

Name : .....

Second Semester B.Sc. Degree Examination, September 2023

First Degree Programme under CBCSS

Mathematics

Complementary Course for Chemistry / Polymer chemistry

MM 1231.2 : MATHEMATICS II – INTEGRAL CALCULUS AND VECTOR DIFFERENTIATION

(2021 Admission Onwards)

Time : 3 Hours

Max. Marks : 80

SECTION – A

Answer **all** questions.

1. Give an example of a proper rational function.

2. Evaluate: $\int \sin 7x \cos 3x \, dx$.

3. Write the formula for integration by parts.

4. What is a surface of revolution?

5. Write the second – order model of the period T of a simple pendulum.

6. Find the parametric equations for the surface generated by revolving the curve $y = \frac{1}{x}$ about the x – axis.

7. Write the conversion formulas for spherical coordinate system to rectangular coordinate system.

8. If $r(t) = t^2 i + e^t j - (2\cos \pi t)k$, find $r'(t)$.

9. Define the gradient of $f(x, y, z)$.

10. Evaluate $\int_0^2 (2ti + 3t^2 j)\, dt$.

(10 × 1 = 10 Marks)

SECTION – B

Answer any **eight** questions.

11. Evaluate : $\int \tan^2 x \sec x \, dx$.

12. Solve the initial value problem $\frac{dy}{dx} = \cos x, y(0) = 1$.

13. Using integration by parts, compute : $\int x^2 \sqrt{x-1}\, dx$.

14. Estimate : $\int \frac{e^{\tan^{-1} x}}{1+x^2} dx$.

15. Compute : $\int_0^2 x(x^2+1)^3\, dx$

16. Find the volume of the solid generated when the region enclosed by $y = \sqrt{x}$, $y = 2$ and $x = 0$ is revolved about the y – axis.

17. Find the area of the region enclosed by $x = y^2$ and $y = x - 2$.

18. Use polar coordinates to compute $\int_{-1}^{1} \int_{0}^{\sqrt{1-x^2}} (x^2 + y^2)^{3/2}\, dy dx$.

19. Find the partial derivatives of $r = ui + vj + (4 - u^2 - v^2)k$.

20. Find the polar coordinates of the point whose rectangular coordinates are $(-2, -2\sqrt{3})$.

21. Let $f(x,y) = xy$. Find $D_u f(1, 2)$ for the unit vector $u = \frac{\sqrt{3}}{2}i + \frac{1}{2}j$.

22. Find the directional derivative of $f(x,y,z) = x^2y - yz^3 + z$ at the point (1, -2, 0) in the direction of the vector $a = 2i + j - 2k$.

(8 × 2 = 16 Marks)

SECTION – C

Answer any **six** questions.

23. Separate $\frac{2x+4}{x^3 - 2x^2}$ into partial fractions.

24. Evaluate : $\int_0^1 \tan^{-1} x \, dx$.

25. Find the total area between the curve $y = 1 - x^2$ and the x – axis over the interval [0, 2].

26. Find the volume of the solid generated when the region between the graphs of the equations $f(x) = \frac{1}{2} + x^2$ and $g(x) = x$ over the interval [0,2] is revolved about the x – axis.

27. Prove : $\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \ldots$

28. Estimate $\iint_A \sin\theta \; dA$ where R is the region in the first quadrant that is outside the circle $r = 2$ and inside the cardioid $r = 2(1 + \cos\theta)$.

29. Find $\iint_R (2x - y^2)\, dA$ over the triangular region R enclosed between the lines $y = -x + 1$, $y = x + 1$ and $y = 3$.

30. Compute the volume of the solid that is bounded above by the plane $z = 4 - x - y$ and below by the rectangle $R = [0,1] \times [0, 2]$.

31. Evaluate $\int_1^3 \int_2^4 (40 - 2xy)\, dy\, dx$.

(6 × 4 = 24 Marks)

SECTION – D

Answer any **two** questions.

32. (a) Find the area of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

(b) Evaluate : $\int e^x \cos x dx$

33. (a) Calculate the area of the surface that is generated by revolving the portion of the curve $y = x^2$ between $x = 1$ and $x = 2$ about the y- axis.

(b) Use cylindrical shells to find the volume of the solid generated when the region under $y = x^2$ over the interval [0, 2] is revolved about the line $y = -1$.

34. Sketch the graph $r = a(1 - \cos\theta)$ in polar coordinates, assuming a to be a positive constant.

35. (a) A heat – seeking, particle is located at the point (2,3) on a flat metal plate whose temperature at (x,y) is $T(x,y) = 10 - 8x^2 - 2y^2$. Find an equation for the trajectory of the particle if it moves continuously in the direction of maximum temperature increase.

(b) Evaluate : $\int_0^1 \int_{-x}^{x^2} y^2 x\, dy\, dx$.

(c) Estimate : $\int_0^{\pi/3} \int_0^{\cos y} x \sin y\, dx\, dy$.

(2 × 15 = 30 Marks)
