(Pages : 4)

R – 3105

Reg. No. : ....

Name : ....

Second Semester B.Sc. Degree Examination, September 2023

First Degree Programme under CBCSS

Statistics

Complementary Course for Mathematics

ST 1231.1 : PROBABILITY AND RANDOM VARIABLES

(2018 – 2021 Admission)

Time : 3 Hours

Max. Marks : 80

SECTION – A

Answer **all** questions. **Each** question carries **1** mark.

1. A coin is tossed until for the first time head appears. Write down the sample space.

2. Define equally likely events.

3. What are the axioms of probability?

4. Describe descrete random variables.

5. State multiplication theorem on probability.

6. Give an example of independent events.

7. The probability function of a descrete random variable is given below. Examine whether it is a probability mass function or not.

x:	0	1	2	3
P(x=x):	0	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$

8. Define expectation of a random variable.

P.T.O.

9. Describe moment generating function.

10. Mention any two properties of characteristic function.

(10 × 1 = 10 Marks)

SECTION – B

Answer any **eight** questions. **Each** question carries **2** marks.

11. Describe the law of statistical regularity.

12. If A and B are two independent events, examine whether $\overline{A}$ and $\overline{B}$ are independent.

13. If A and B are two events such that $P(A)=0.2; P(B)=0.3$ and $P(A \cap B)=0.1$, find probability of

    (a) At least one occur

    (b) Only A but not B occur.

14. Describe conditional probability.

15. Let X be a random variable with probability mass function.

x:	0	1	2	3	4
P(x=x):	$1/6$	$1/2$	$3/30$	$2/30$	$1/6$

Find

(a) $P(x \leq 1)$; (b) $P(0 < x < 3)$

16. Define distribution function and give any one of it properties.

17. Find the characteristics function of x with probability function. $f(x)=\frac{1}{2^x}, x=1,2,...$

18. Mention any two drawbacks of moment generating function.

19. Give the mathematical definition of probability. Give one of its drawbacks.

20. The joint probability density function of random variables (x, y) is $f(x,y)=e^{-(x+y)}, x>0, y>0$. Find the marginal distribution of x.

21. If x is a random variable and a and b are constants, show that $E(ax+b)=aE(x)+b$.

22. The moment generating function of a random variable x is $\left(1-t/\theta\right)^{-1}$. Find the first two raw moments.

(8 × 2 = 16 Marks)

SECTION – C

Answer any **six** questions. **Each** question carries **4** marks.

23. State and prove addition theorem on probability of two events.
24. Prove or disprove pairwise independence imply mutual independence.
25. State and prove Baye's theorem.
26. Two cards are drawn from a pack of well shuffled cards. Find the probability of getting
    (a) Two aces;
    (b) Two diamond cards.
27. Describe random variables. Find the distribution function of the random variable Number of heads occurred in five tosses of an unbiased coin.
28. If x and y are two random variable such that $f(x,y)=\frac{1}{8}(6-x-y),\ 0<x\leq 2;$ $2<y\leq 4$. Find the marginal distributions of x and y.
29. Describe conditional mean and conditional variance. Show that $E\left[E\left(x/y\right)\right]=E(x)$.
30. State and establish cauchy–Schwatz in equality.
31. Find the moment generating function of x with probability mass function $P(x=x)\frac{e^{-\lambda}}{x!}\lambda^{x},\ x=0,1,2\ldots$ and hence find its first two raw moments.

(6 × 4 = 24 Marks)

SECTION – D

Answer any **two** questions. **Each** question carries **15** marks.

32. (a) For n events $A_1, A_2, \ldots, A_n$, show that
    (i) $P(A_1 \cap A_2 \cap \ldots \cap A_n) \geq P(A_1)+P(A_2)+\ldots+P(A_n)-(n-1)$
    (ii) $P(A_1 \cup A_2 \cup \ldots \cup A_n) \leq P(A_1)+P(A_2)+\ldots+P(A_n)$
    (b) For any three events A, B and C show that
    (i) $P\left[A\cup B/C\right]=P\left(A/C\right)+P\left(B/C\right)-P\left[A\cap B/C\right]$
    (ii) $P[(A\cap B)/C]+P\left[(A\cap\overline{B})/C\right]=P\left(A/C\right)$

33. (a) Two persons A and B alternatively cut a pack of cards and the pack is shuffled after each cut. If A starts and continued the game until one cuts a diamond card. What are the respective chances of A and B first cutting a diamond card.

(b) Three persons A, B and C appeared a competative examination. their chances of selection is in the ratio 4:2:3 respectively. A if selected will introduce a new system with probability 0.3. The probability of B and C doing the same are respectively 0.5 and 0.8. If the new system is introduced, what is the probability that

(i) A is selected

(ii) B is selected

34. (a) A continuous random variable x has the probability function $f(x)=3x^2, 0<x\leq 1$. Find

(i) $E(x)$;

(ii) $V(x)$;

(iii) Value of a such that $P(x\leq a)=P(x>a)=\frac{1}{2}$

(b) With usual notations show that $E\left[v\left(x/y\right)\right]+V\left[E\left(x/y\right)\right]=V(x)$.

35. (a) Given the following bivariate probability function. find

(i) Marginal distributions of x and y.

(ii) Conditional distribution of x given y=2.

x \ y	-1	0	1
0	$\frac{1}{15}$	$\frac{2}{15}$	$\frac{1}{15}$
1	$\frac{3}{15}$	$\frac{2}{15}$	$\frac{1}{15}$
2	$\frac{2}{15}$	$\frac{1}{15}$	$\frac{2}{15}$

(b) Define independence of two random variables. Show that $V(ax+by)=a^2V(x)+b^2V(y)$ if x and y are independent.

(2 × 15 = 30 Marks)
