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Reg. No. : ....

Name : ....

Second Semester B.Sc. Degree Examination, September 2023

First Degree Programme under CBCSS

Statistics

Complementary Course For Mathematics

ST 1231.1 : PROBABILITY AND RANDOM VARIABLES

(2022 Admission)

Time : 3 Hours Max. Marks : 80

SECTION – A

Answer **all** questions. **Each** question carries **1** mark.

1. Define sample space and sample points.

2. What do you mean by equally likely events?

3. Give the classical definition of probability.

4. Using the axiomatic definition of probability indicate the validity of the statement $P(A^c) = 1 - P(A)$.

5. If A and B are independent, then prove that $P(A/B) = P(A/B^c)$.

6. Define a random variable with an example.

7. Give the definition of the distribution function of a two dimensional random vector.

8. State the multiplication theorem on expectation.

P.T.O.

9. Define conditional variance of a random variable.

10. Define moment generating function of a random variable.

(10 × 1 = 10 Marks)

SECTION – B

Answer any **eight** questions. **Each** question carries **2** marks.

11. Distinguish between random experiments and deterministic experiments with examples.

12. Define statistical regularity.

13. Let A and B be two events which are not mutually exclusive and if $P(A)=\frac{1}{4}$, $P(B)=\frac{2}{5}$ and $P(A\cap B)=\frac{3}{20}$. Then find the probability that

    (a) Either one of them will happen and

    (b) None of them will happen.

14. Give the frequency definition of probability and mention its limitations.

15. Define the probability space and probability measure associated with a random experiment.

16. State Baye's theorem.

17. Distinguish between discrete and continuous random variables.

18. Define a distribution function and mention its properties.

19. Find the conditional pdf of X/Y, if the joint pdf of (X,Y) is $f(x,y)=2-x-y, 0<x,y<1$.

20. Let $f(x)=|x|$ *for* $|x|\leq c$ and $f(x)=0$ elsewhere. Find the value of c, so that $f(x)$ is a probability density functions (pdf).

21. Mention the important properties of mathematical expectation.

22. Define the coefficient of linear correlation between two random variables.

(8 × 2 = 16 Marks)

SECTION – C

Answer any **six** questions. **Each** question carries **4** marks.

23. Given $P(A) = P(B) = P(C) = 0.4$, $P(A \cap B) = P(A \cap C) = P(B \cap C) = 0.2$ and $P(A \cap B \cap C) = .01$, find the probabilities of

    (a) At least one of the events and

    (b) None of the events happen.

24. State and prove the multiplication rule of probability.

25. Prove or disprove: Mutual independence of three events implies pairwise independent.

26. If the joint pdf of X and Y is $p(x, y) = \frac{1}{22}(2x + 3y), x = 0,1$ and $y = 1,2$ what is the joint distribution function?

27. The joint pdf of X and Y is $f(x, y) = 2 - x - y, 0 < x, y < 1$. Find

    (a) The marginal pdfs of X and Y

    (b) Conditional pdf of (X / Y).

28. If $f(x) = e^{-x}, x \geq 0$, find the pdf of $Y = \sqrt{X}$.

29. If X is a random variable with pdf $f(x) = \frac{x}{6}, x = 1,2,3$; find $E(X + 2)^2$.

30. Show that $E(X^2) \geq (E(X))^2$.

31. State and prove the Cauchy–Schwartz inequality.

(6 × 4 = 24 Marks)

SECTION – D

Answer any **two** questions. **Each** question carries **15** marks.

32. (a) Give the axiomatic definition of probability.

(b) The probability that a man will be alive for 25 years is 3/5 and that his wife will be alive for 25 years is 2/3. Find the probability that

(i) both will be alive,

(ii) only wife will be alive,

(iii) at least one of them will be alive and

(iv) none of them will be alive.

33. (a) State and prove the addition law of probability of two events. State the law for three events.

(b) There are 2 bags, each containing respectively 4 white and 3 black balls and 3 white and 7 black balls. A bag is chosen at random and a ball is drawn from it. Find the probability that the ball is white. If the ball is white, what is the probability that it is from the first bag?

34. (a) Define the concept of transformation of one dimensional random variables.

(b) Find the characteristic function and hence obtain the mean and variance of the random variable with a pdf $f(x) = \theta e^{-\theta x}$, $x \geq 0, \theta > 0$.

35. (a) A continuous random variable X has the pdf $f(x) = 2x, 0 < x \leq 1,$ and 0 elsewhere. Find

(i) $F(x)$

(ii) $P\left(X \leq \frac{1}{2}\right)$ and

(iii) $P(\frac{1}{4} \leq X \leq \frac{3}{4})$.

(b) Examine the independence of the random variables with joint probability density function $f\,x,y(x,y) = 6(x - y), 0 < y < x < 1$. Also compute $E(Y/X)$ and $V(Y/X)$.

(2 × 15 = 30 Marks)
