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Reg. No. : ....

Name : ....

Fourth Semester B.Sc. Degree Examination, July 2023

First Degree Programme under CBCSS

Mathematics

Complementary Course for Chemistry/Polymer Chemistry

MM 1431.2 : MATHEMATICS IV — DIFFERENTIAL EQUATIONS, VECTOR CALCULUS AND ABSTRACT ALGEBRA

(2021 Admission)

Time : 3 Hours Max. Marks : 80

PART – A

Answer **all** questions. Each carries **1** mark.

1. Write the order and degree of the differential equation $\left(\frac{dy}{dx}\right)^3 + 2y = \left(\frac{d^2y}{dx^2}\right)^2$.

2. Find an integrating factor of the differential equation $(x+1)\frac{dy}{dx} - y = e^{3x}(x+1)^2$.

3. Write the general form of Cauchy's homogenous linear equation.

4. Find the Wronskian of e^x and e^{-x}.

5. Define the inverse square field of a radius vector $\overline{x}$.

P.T.O.

6. If C is a piecewise smooth curve from (1, 2, 3) to (4, 5, 6), then $\int_C dx + 2dy + 3dz =$ ________.

7. State Gauss's law for inverse square fields.

8. Find all solutions of $x +_{12} x = 2$ in z_{12}.

9. Find the order of the subgroup of z_6 generated by 3.

10. Describe all units in the ring Q.

(10 × 1 = 10 Marks)

PART – B

Answer **any eight** questions. Each carries **2** marks.

11. Solve $\frac{dy}{dx} = e^{3x-2y} + x^2 e^{-2y}$.

12. Check whether the differential equation $(y \cos x + 1)\, dx + \sin x\, dy = 0$ is exact.

13. Solve $\frac{d^4y}{dx^4} + 13\frac{d^2y}{dx^2} + 36y = 0$.

14. Transform the differential equation $x^2\frac{d^2y}{dx^2} + 3x\frac{dy}{dx} + y = \frac{1}{(1-x)^2}$ into a linear equation with constant coefficients.

15. Find the divergence of $\bar{v} = xyz\hat{i} + 3x^2y\hat{j} + (xz^2 - y^2z)\hat{k}$ at the point $(2, -1, 1)$.

16. Evaluate $\int_C (1 + xy^2)\, ds$, where $c : \bar{r}(t) = t\hat{i} + 2t\hat{j},\ 0 \le t \le 1$.

17. Find $\iint_\sigma (x^2 + y^2 + z^2)\, ds$, where σ is the sphere of radius 2 centred at the origin.

18. Using divergence theorem, find the outward flux of the vector field $\overline{F}(x, y, z) = z\hat{k}$ across the sphere $x^2 + y^2 + z^2 = a^2$.

19. Determine whether the binary operation $*$ defined on z^+ by $a * b = a^b$ is associative.

20. Write the group table of z_4.

21. Compute M_{σ^2} if $\mu = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 5 & 2 & 4 & 3 & 1 & 6 \end{pmatrix}$ and $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 1 & 4 & 5 & 6 & 2 \end{pmatrix}$.

22. Define ring. Give an example of a ring with unity.

(8 × 2 = 16 Marks)

PART – C

Answer **any six** questions. Each carries **4** marks

23. Solve $\left[x \tan\left(\frac{y}{x}\right) - y \sec^2\left(\frac{y}{x}\right)\right]dx - x\sec^2\left(\frac{y}{x}\right)dy = 0$,

24. Solve $y = 2px - p^3$.

25. Using the method of variation of parameters solve $\frac{d^2y}{dx^2} + 4y = \tan 2x$.

26. Prove : $curl\,(\varphi \overline{F}) = \varphi\, curl\, \overline{F} + \nabla\varphi \times \overline{F}$.

27. Find the work done by the conservative field $\overline{F}(x, y) = e^y\hat{i} + xe^y\hat{j}$ on a particle that more from (1, 0) to (–1, 0) along a semicircular path.

28. Apply Green's theorem to evaluate $\int_C (2x^2 - y^2)\,dx + (x^2 + y^2)\,dy$ where C is the boundary of the area enclosed by the x-axis and the upper half of the circle $x^2 + y^2 = a^2$.

29. Show that $(Z, +)$ is isomorphic to $(2Z, +)$, where + is the usual addition.

30. Find all subgroups of Z_{12} and draw its subgroup diagram.

31. If R is a ring with additive identity o, then for any $a, b \in R$, prove that

(a) $oa = ao = 0$

(b) $a(-b) = (-a)b = -(ab)$

(6 × 4 = 24 Marks)

PART – D

Answer **any two** questions. Each carries **15** marks

32. (a) Solve $(3y + 2x + 4)dx - (4x + 6y + 5)dy = 0$.

(b) Solve $(xy^3 + y)dx + 2(x^2y^2 + x + y^4)dy = 0$

33. (a) Find the orthogonal trajectory of the cardiods $r = a(1 - \cos\theta)$.

(b) Solve $(D-2)^2 y = 8(e^{2x} + \sin 2x + x^2)$.

34. Varity Stoke's theorem for the vector field $\overline{F} = (2x - y)\hat{i} - yz^2\hat{j} - y^2z\hat{k}$ over the upper half surface of $x^2 + y^2 + z^2 = 1$, bounded by its projection on the xy-plane.

35. (a) Show that if G is a finite group with identity e and with an even number of elements, then there is an element $a \neq e$ in G such that $a * a = e$.

(b) Let $(R_1 +)$ be an abelian group. Show that $(R, +, \cdot)$ is a ring if we define $ab = 0, \forall\ a, b \in R$.

(2 × 15 = 30 Marks)
