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R – 2350

Reg. No. : ....

Name : ....

Fourth Semester B.Sc. Degree Examination, July 2023

First Degree Programme under CBCSS

Mathematics

Complementary Course for Chemistry/Polymer Chemistry

MM 1431.2 : MATHEMATICS IV – DIFFERENTIAL EQUATIONS, VECTOR CALCULUS AND ABSTRACT ALGEBRA

(2018-2020 Admission)

Time : 3 Hours Max. Marks : 80

PART – A

Answer **all** questions. Each carries **1** mark.

1. Write the form of exact first degree first order ODE.

2. Say true or false: The Bernoulli's equation is a linear equation.

3. What is the most general standard form is Higher-degree first-order equation.

4. The Legendre's equation has the form ____________.

5. Say true or false: The line integral depends on the end-points A and B but not on the path C joining them.

6. If P is any point on the path of integration that lies between the path's end-points A and B then $\int_A^B a \cdot dr =$ ____________.

7. Let V is a small volume enclosing P and S is its bounding surface. If φ is a scalar field and a is a vector field then at any point P, $\nabla \cdot a =$ ____________.

P.T.O.

8. In a group $\mathcal{G}$, $(U \bullet V \bullet \cdots \bullet Y \bullet Z)^{-1} =$ ______________.

9. If $X^m = I$, then m is called the ______________ of the element X in $\mathcal{G}$.

10. Define homomorphism of a group $\mathcal{G}$.

(10 × 1 = 10 Marks)

PART – B

Answer any **eight** questions. These questions carry **2** marks each.

11. Solve $\frac{dy}{dx} = x + xy$.

12. Solve $\frac{dy}{dx} = (x + y = 1)^2$.

13. Find the complementary function of the equation $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = e^x$.

14. Find a solution of $(x^2 + x)\frac{dy}{dx}\frac{d^2y}{dx^2} - x^2y\frac{dy}{dx} - x\left(\frac{dy}{dx}\right)^2 = 0$.

15. Solve $\frac{dy}{dx} + 2xy = 4x$.

16. Find an expression for the angular momentum of a solid body rotating with angular velocity ω about an axis through the origin.

17. Find the volume enclosed between a sphere of radius a centred on the origin and a circular cone of half-angle α with its vertex at the origin.

18. Reduce $\int_C a \cdot dr$ to a set of scalar integrals by writing the vector field a in terms of its Cartesian components as $a = a_x i + a_y j + a_z k$, where a_x, a_y, a_z are each (in general) functions of x, y, z.

19. When do you say that a plane region R is doubly connected?

20. Consider the ordered set of six distinct objects $\{a\ b\ c\ d\ e\ f\}$. Suppose ϕ is the permutation [4 5 3 6 2 1] and θ is the permutation [2 5 6 1 4 3]. Then find $\phi \bullet \theta\{a\ b\ c\ d\ e\ f\}$.

21. Define isomorphism of groups.

22. Write three properties of the subgroups of a group $\mathcal{G}$.

(8 × 2 = 16 Marks)

PART – C

Answer any **six** questions. These questions carry **4** marks each.

23. Solve $x\frac{dy}{dx}+3x+y=0$.

24. Solve $(x^3+x^2+x+1)p^2-(3x^2+2x+1)yp+2xy^2=0$.

25. Solve $\frac{d^2y}{dx^2}+2\frac{dy}{dx}=4x$.

26. Evaluate the line integral $I=\oint_C xdy$, where C is the circle in the xy-plane defined by $x^2+y^2=a^2$, $z=0$.

27. Evaluate the line integral $I=\oint_C[(e^x y+\cos x\sin y)dx+(e^x+\sin x\cos y)dy]$ around the ellipse $x^2/a^2+y^2/b^2=1$.

28. Find the vector area of the surface of the hemisphere $x^2+y^2+z^2=a^2$, $z\geq 0$, by evaluating the line integral $S=\frac{1}{2}\oint_C r\times dr$ around its perimeter.

29. Let $\Phi:\mathcal{G}\rightarrow\mathcal{G}'$ be a homomorphism of $\mathcal{G}$ into $\mathcal{G}'$; then show that the set of elements $\mathcal{K}$ in $\mathcal{G}$ that are mapped onto the identity I′ in $\mathcal{G}'$ forms a subgroup of $\mathcal{G}$.

30. Show that the traces of equivalent matrices are equal.

31. For the hydrogen molecule consists of two atoms H of hydrogen, what are different sets of operations rotations, reflections, and inversions.

(6 × 4 = 24 Marks)

PART – D

Answer any **two** questions. These questions carry **15** marks each.

32. (a) A house-buyer borrows capital B from a bank that charges a fixed annual rate of interest R%. If the loan is to be repaid over Y years, at what value should the fixed annual payments P, made at the end of each year, be set? For a loan over 25 years at 6%, what percentage of the first year's payment goes towards paying off the capital?

(b) Two electrical circuits, both of negligible resistance, each consist of a coil having self-inductance L and a capacitor having capacitance C. The mutual inductance of the two circuits is M. There is no source of e.m.f. in either circuit. Initially the second capacitor is given a charge CV_0, the first capacitor being uncharged, and at time $t = 0$ a switch in the second circuit is closed to complete the circuit. Find the subsequent current in the first circuit.

33. (a) Solve $(1-x^2)\frac{d^2y}{dx^2} - 3x\frac{dy}{dx} - y = 1$.

(b) Use the variation-of-parameters method to solve $\frac{d^2y}{dx^2} + y = \operatorname{cosec} x$ subject to the boundary conditions $y(0) = y(\pi/2) = 0$.

34. Evaluate the line integral $I = \int_A^B a \cdot dr$ where $a = (xy^2 + z)i + (x^2y + 2)j + xk$, A is the point (c, c, h) and B is the point (2c, c/2, h), along the different paths

(a) C_1, given by $x = cu$, $y = c/u$, $z = h$,

(b) C_2, given by $2y = 3c - x$, $z = h$. Show that the vector field a is in fact conservative, and find φ such that $a = \nabla\varphi$.

35. If n_μ is the dimension of the μth irrep of a group G then show that $\sum_\mu n_\mu^2 = g$, where g is the order of the group.

(2 × 15 = 30 Marks)