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Reg. No. : ....

Name : ....

Fourth Semester B.Sc. Degree Examination, July 2023

First Degree Programme under CBCSS

Mathematics

Complementary Course for Physics

MM 1431.1 MATHEMATICS IV – COMPLEX ANALYSIS, SPECIAL FUNCTIONS AND PROBABILITY THEORY

(2018 – 2020 Admissions)

Time : 3 Hours Max. Marks : 80

SECTION – I

All the first **ten** questions are compulsory. They carry **1** mark each.

1. Find the real part of $\frac{\overline{Z}}{Z}$.

2. Define a harmonic function.

3. Find the residue of $f(z) = \frac{z^2 - 1}{z^2 + z}$ at $z = 0$.

4. Show that $\int_C \frac{z^3}{z-2} dz = 0$ where C is the circle $|z| = 1$.

5. State the recurrence relation for Gamma function.

6. Define $\beta(p, q)$.

7. A bag contains 2 white and 4 red balls. Find the probability of getting a red ball.

P.T.O.

8. If the mean and variance of the Binomial distribution are 50 and 5 respectively. Find the number of trials.

9. What is the probability that a non-leap year selected at random will contain 53 Sundays?

10. A card is drawn from a well shuffled deck of cards. What is the probability of getting a king or a club?

(10 × 1 = 10 Marks)

SECTION – II

Answer **any eight** questions. Each question carries **2** marks.

11. Define an entire function.

12. Use Cauchy - Riemann equations to prove that the function $f(z) = x^2 - y^2 + 2i\ xy$ is analytic.

13. At what points in the z-plane at which the mapping $f(z) = 5z^5 - 5z$ is not conformal.

14. Find the singular points of the function $\dfrac{z+1}{z^3(z^2+1)}$

15. Find R (0) for $f(z) = \dfrac{\cos z}{z}$.

16. Express the integral $\int_0^1 \dfrac{x^2}{(1-x^5)^{\frac{1}{2}}}\,dx$ in terms of Beta function .

17. Evaluate $\int_0^\infty e^{-x^3}\,dx$.

18. Show that $\Gamma(n) = \int_0^1 \left(\log \dfrac{1}{x}\right)^{n-1} dx$.

19. What is the probability that a leap year selected at random will contain 53 Sundays?

20. Two students are working independently on the same problem. If the first student has the probability 3/5 of solving it and the second has the probability 1/5 of solving it. Find the probability that the problem is solved

21. Check whether $f(x)=\frac{3}{4}x(2-x)$ for $0\le x\le 2$ is a probability density function.

22. A probability function $P(x)$ is given by $P(x)=k\left(\frac{2}{3}\right)^x$, $x=1, 2, 3,...$; Find k.

(8 × 2 = 16 Marks)

SECTION – III

Answer **any six** questions. Each question carries 4 marks.

23. Prove that $\frac{d}{dz}[f(z)g(z)]=f(z)g'(z)+f'(z)g(z)$.

24. Expand $f(z)=\frac{2-z}{1-z^2}$ as a Laurent's series valid for $|z|>1$.

25. Discuss the conformality of $w = e^z$ and show that it transforms vertical line into a circle.

26. Show that $\Gamma(1/2)=\sqrt{\pi}$.

27. Show that $\beta(p, q)=\beta(q, p)$.

28. Two persons A and B throw a die alternatively till one of them gets a '4' and wins the game. Find their respective probabilities of winning, if A starts the game.

29. In a section of 20 students half of them got first class and the rest second class. In another section of 30 students, these ratios are in the ratio 1:2. If a candidate is selected at random and he happens to be a first class, What is the probability that he comes from the second section?

30. Suppose four chips are marked 1, 2, 3, 4. Two chips are drawn at random without replacement. Let X stand for the sum of the numbers on the two chips, find the probability density function of X.

31. Find the mean and variance of a random variable x which takes the values 1, 2, 3, 4, 5 with respective probabilities $\frac{1}{10}, \frac{2}{10}, \frac{4}{10}, \frac{2}{10}, \frac{1}{10}$.

(6 × 4 = 24 Marks)

SECTION – IV

Answer **any two** questions. Each question carries **15** marks.

32. (a) Derive Cauchy-Riemann equations.

(b) Evaluate $\int_0^{2\pi} \frac{d\theta}{5+4\cos\theta}$.

33. (a) Prove the relation between Beta and Gamma function : $\beta(p, q)=\frac{\Gamma p\, \Gamma q}{\Gamma(p+q)}$.

(b) Show that $\beta(p+1, q)=\frac{p}{p+q}\beta(p, q)$.

34. (a) Obtain the recurrence relation for the central moment of binomial distribution.

(b) It is known that 5% of certain items are defective. Using binomial distribution, find the probability that a random sample of 8 such items will contain less than 2 defective items.

35. (a) Show that of a Poisson distribution with parameter λ, mean = variance = λ.

(b) If X is a Poisson variate such that $P[X=1]=0.3$ and $P[X-2]=0.2$, find the value of $P[X=0]$ and $P[X=3]$.

(2 × 15 = 30 Marks)