



Reg. No. : .....

Name : .....

**Fourth Semester B.Sc. Degree Examination, July 2023**

**First Degree Programme under CBCSS**

**Mathematics**

**Complementary Course for Physics**

**MM 1431.1 — MATHEMATICS IV – FOURIER SERIES, COMPLEX  
ANALYSIS AND PROBABILITY THEORY**

**(2021 Admission)**

Time : 3 Hours

Max. Marks : 80

**PART – A**

Answer **all** questions. Each question carries **1** mark.

1. State the Cauchy's Integral formula.
2. Find the real part of  $f(z) = z^2 + 3z$ .
3. Find the Fourier sine series of the function  $f(x) = x$ ,  $0 \leq x < \pi$ .
4. Find the residue of  $e^{\frac{1}{z}}$  at  $z = 0$ .
5. A pack of 52 cards two are drawn at random. Find the probability that both are aces.
6. State Baye's theorem.



7. State Cauchy – Riemann equation.
8. If  $F(s)$  is the Fourier transform of  $f(x)$  then what is the Fourier transform of  $f(x - a)$  where  $a$  is any real number.
9. Given an example for an even function.
10. Out of 50 members of a club in how many ways can a president, vice president and a secretary can be chosen.

(10 × 1 = 10 Marks)

### PART – B

Answer any **eight** questions. Each question carries **2** marks.

11. Find all the values of  $(-i)^{\frac{1}{3}}$ .
12. Define periodic function and give an example.
13. Distinguish between discrete and continuous random variables.
14. State Laurent's theorem.
15. Find the probability that a single card drawn from a shuffled deck of cards will be either a diamond or king.
16. Evaluate  $\oint_C \frac{z+1}{z^2} dz$ , where  $C$  is the unit circle.
17. Expand  $f(x) = (x - 1)^2$ ,  $0 < x < 1$  in a Fourier series of sine terms only.
18. Determine residue of  $f(z) = \frac{\cos z}{z^4}$ , at  $z = 0$ .
19. Find the Fourier sine transform of  $f(x) = e^{-ax}$ , where  $a > 0$ .
20. A random variable  $X$  has the density function  $f(x) = kx$  for  $2 \leq x \leq 5$ . Find  $k$ .
21. Find the Binomial distribution whose mean is 9 and variance is 6.
22. Find the half range cosine series of  $f(x) = x$ ,  $0 < x < \pi$ .

(8 × 2 = 16 Marks)



## PART – C

Answer any **six** questions. Each question carries **4** marks.

23. Expand  $\frac{1}{z^2 - 3z + 2}$  in Laurent's series valid in the region  $1 < |z| < 2$ .
24. Find the mean and variance of a random variable  $X$  which takes the values 0, 1, 2, 3 with respective probabilities  $\frac{1}{8}, \frac{3}{8}, \frac{3}{8}, \frac{1}{8}$ .
25. Find the Fourier series of the function  $f(x) = |x|$ ,  $-\pi \leq x \leq \pi$ .
26. Evaluate  $\int_C \tan z \, dz$  where  $C$  is  $|z| = 2$ .
27. Find the Fourier Transform of  $e^{-x^2}$ .
28. Prove that  $U = x^3 - 3xy^2 - 5y$  is harmonic and find its harmonic conjugate.
29. Find the Fourier sine transform of  $\frac{e^{-ax}}{x}$ .
30. If the probability of a bad reaction from a certain injection is 0.001, determine the chance that out of 2000 individuals more than two will get a bad reaction.
31. A problem in mathematics is given to three students  $A$ ,  $B$  and  $C$  whose chances of solving it are  $\frac{1}{2}$ ,  $\frac{1}{3}$  and  $\frac{1}{4}$  respectively. What is the probability that the problem will be solved?

(6 × 4 = 24 Marks)

## PART – D

Answer any **two** questions. Each question carries **15** marks.

32. (a) There are 3 bags: first containing 1 white, 2 red, 3 green balls; second 2 white, 3 red, 1 green balls and third 3 white, 1 red, 2 green balls. Two balls are drawn from a bag chosen at random. These are found to be one white and one red. Find the probability that the balls so drawn came from the second bag.
- (b) In a normal distribution, 31% of the items are under 45 and 8% are over 64. Find the mean and S.D of the distribution.



33. Find the Fourier series of the function  $f(x) = \begin{cases} -\pi & \text{if } -\pi < x < 0 \\ x & \text{if } 0 < x < \pi \end{cases}$  and Compute the sum  $1 + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \dots$

34. (a) Use Cauchy's integral formula to evaluate  $\int_C \frac{z+1}{z^4 + 2iz^3} dz$  where  $C$   $|z| = 1$ .

(b) Evaluate  $\int_C \bar{z} dz$  where  $C$  is given by  $x = 3t$ ,  $y = t^2$ ,  $1 \leq t \leq 4$ .

35. (a) Evaluate  $\int_0^{2\pi} \frac{\cos 3\theta}{5 - 4 \cos \theta} d\theta$ .

(b) Suppose two dice are thrown. If  $X$  denote the sum of the numbers on the dice, find the probability density function of  $X$ .

**(2 × 15 = 30 Marks)**

