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**S – 2704**



Reg. No. : ..............................

Name : ..................................

**First Semester B.Sc. Degree Examination, January 2024**

**First Degree Programme under CBCSS**

**Mathematics**

**Complementary Course for Chemistry and Polymer Chemistry**

**MM 1131.2 : Mathematics I – DIFFERENTIAL CALCULUS AND SEQUENCE AND SERIES**

**(2021 Admission Onwards)**

Time : 3 Hours

Max. Marks : 80

SECTION – A

Answer **all** questions.

1. If $f(x) = 3x^4 - 2x^3 + x^2 - 4x + 2$, then compute $f^{(5)}(x)$.

2. Find $\frac{d}{dx}\left(\ln\left[x^2+1\right]\right)$.

3. Find : $\lim_{x \to 2} \frac{x^2-4}{x-2}$.

4. Define an inflection point of a continuous function *f*.

5. Let $f(x, y) = y^2 e^x + y$. Evaluate $f_{xyy}$.

6. Write the steps to find the absolute extrema of a continuous function *f* of two variables on a closed and bounded set *R*.

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7. Find $\frac{dy}{dx}$ if $x^3 + y^2x - 3 = 0$.

8. State the Squeezing theorem for sequences.

9. Verify whether the series $\sum_{k=1}^{\infty} \frac{k}{2^k}$ converges.

10. Write the Bessel function $J_0(x)$ using sigma notation.

**(10 × 1 = 10 Marks)**

SECTION – B

Answer any **eight** questions.

11. Let $f(x) = \begin{cases} \frac{1}{x+2}, & x < -2 \\ x^2 - 5, & -2 < x \le 3 \\ \sqrt{x+13}, & x > 3 \end{cases}$. Estimate $\lim_{x \to -2} f(x)$.

12. Find $\frac{dy}{dx}$ if $y = x \sin x$.

13. Compute $\frac{dw}{dt}$ if $w = \tan x$ and $x = 4t^3 + t$.

14. Find : $\lim_{x \to 0} \frac{e^x - 1}{x^3}$.

15. Obtain all critical points of $f(x) = x^3 - 3x + 1$.

16. Find the two $x$ – intercepts of the function $f(x) = x^2 - 5x + 4$ and confirm that $f'(c) = 0$ at some point $c$ between those intercepts.

17. State the Rolle's theorem.

18. Evaluate $f_x(1, 3)$ and $f_y(1, 3)$ where $f(x, y) = 2x^3y^2 + 2y + 4x$.

19. State the chain rules for partial derivatives.

20. If $w = xy + yz$, $y = \sin x$, $z = e^x$, then estimate $\frac{dw}{dx}$.

21. Find the sum of the series $\sum_{k=1}^{\infty}\left(\frac{3}{4^k} - \frac{2}{5^{k-1}}\right)$.

22. Determine whether the series $\sum_{k=1}^{\infty}\frac{1}{\sqrt{k} - \frac{1}{2}}$ converges.

**(8 × 2 = 16 Marks)**

SECTION – C

Answer any **six** questions.

23. Evaluate : $\lim_{x \to +\infty}\frac{\sqrt{x^2+2}}{3x-6}$.

24. Find $y'(x)$ for $y = \frac{x^3 + 2x^2 - 1}{x+5}$.

25. Estimate : (a) $\lim_{x \to 0^+} x \ln x$ (b) $\lim_{x \to \pi/4}(1 - \tan x)\sec 2x$.

26. Find the relative extrema of $f(x) = 3x^5 - 5x^3$.

27. Identify the intervals on which $f(x) = x^3$ is increasing and the intervals on which it is decreasing.

28. Let $f(x, y) = x^2y + 5y^3$. Find the slope of $z = f(x, y)$ in the

(a) $x$ – direction at the point $(1, -2)$

(b) $y$ – direction at the point $(1, -2)$.

29. Show that $u(x, t) = \sin(x - ct)$ is a solution of the one-dimensional wave equation.

30. Locate all relative extrema and saddle points of $f(x, y) = 3x^2 - 2xy + y^2 - 8y$.

31. Find the $n^{th}$ Maclaurin polynomial for $\frac{1}{1-x}$ and express it in sigma notation.

**(6 × 4 = 24 Marks)**

SECTION – D

Answer any **two** questions.

32. Determine whether the function $f(x) = \frac{1}{x^2 - x}$ has any absolute extrema on the interval $(0, 1)$. If so, find them and state where they occur.

33. (a) The length, width and height of a rectangular box are measured with an error of at most 5%. Use a total differential to estimate the maximum percentage error that results in these quantities that are used to calculate the diagonal of the box.

(b) Use the method of Lagrange multipliers to find the dimensions of a rectangle with perimeter $p$ and maximum area.

34. Find the absolute maximum and minimum values of $f(x, y) = 3xy - 6x - 3y + 7$ on the closed triangular region $R$ with vertices $(0, 0)$, $(3, 0)$ and $(0, 5)$.

35. Use the ratio test to determine whether the following series converge or diverge:

(a) $\sum_{k=1}^{\infty} \frac{1}{k!}$ (b) $\sum_{k=1}^{\infty} \frac{(-1)^k 2^k}{k!}$ (c) $\sum_{k=1}^{\infty} \frac{k^k}{k!}$ (d) $\sum_{k=3}^{\infty} \frac{(2k)!}{4^k}$ (e) $\sum_{k=1}^{\infty} \frac{1}{2k-1}$

**(2 × 15 = 30 Marks)**

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