

Reg. No. : .....

Name : .....



First Semester B.Sc. Degree Examination, January 2024

First Degree Programme under CBCSS

Core Course

Mathematics

MM 1141 : METHODS OF MATHEMATICS

(2018 - 2022 Admission)

Time : 3 Hours

Max. Marks : 80

SECTION - I

Answer all question compulsory. Each question carries 1 mark.

1. Find the local approximation of  $f(x) = \sin x$  at  $x_0 = 0$ .
2. The time required for one complete oscillation of a pendulum is called its \_\_\_\_\_
3. The critical points of  $f(x) = x^3 - 3x + 1$  occurs positively at  $x =$  \_\_\_\_\_
4. If all units produced are sold, then the cost function, revenue function and profit function are related by \_\_\_\_\_
5. An interval on which  $f(x) = x^2 - x$  satisfies the hypothesis of Rolle's Theorem is \_\_\_\_\_
6. The smallest possible value of  $f(x) = x + \frac{1}{x}$  is obtained for positive number  $x$  in the interval \_\_\_\_\_

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7. The points of intersection for the circle  $x^2 + y^2 = 4$  and the line  $y = x + 2$  and \_\_\_\_\_ and \_\_\_\_\_
8. A function  $f$  is smooth on  $[a, b]$ , if  $f'$  is \_\_\_\_\_
9. A force  $F(x) = 10 - 2x$  N applied in the positive  $x$ -direction moves on object  $3m$  from  $x = 2$  to  $x = 5$ . Then the workdone by the force on the object is \_\_\_\_\_
10. The parametric equations  $x = \cosh t, y = \sinh t (-\infty < t < +\infty)$  represent the right half of the curve called a \_\_\_\_\_

(10 × 1 = 10 Marks)

### SECTION – II

Answer any **eight** questions. **Each** question carries **2** marks.

11. Suppose that  $x$  and  $y$  differentiable functions of  $t$  and are related by the equation  $y = x^3$ . Find  $\frac{dy}{dt}$  at time  $t = 1$  if  $x = 2$  and  $\frac{dx}{dt}$  at time  $t = 1$ .
12. Find formula for  $\Delta y$  and  $dy$  for  $y = \sqrt{x}$ .
13. Evaluate  $\lim_{x \rightarrow 0} \frac{\sin 2x}{x}$  using L' Hospital's rule.
14. Draw the graph for  $y = x^3 - 3x + 3$ .
15. Define Mean-Value Theorem.
16. Find the two  $x$ -intercepts of the function  $f(x) = x^2 - 5x + 4$ .
17. Write the steps to find the limits of integration for the area between two curves.
18. Find the volume of the solid generated when the region enclosed by  $y = \sqrt{x}, y = 2$  and  $x = 0$  is resolved about the  $y$ -axis.

19. A spring exerts a force of 5 N when stretched 1 m beyond its natural length. How much work is required to stretch the spring 1.8 m beyond its natural length.
20. The face of a dam is a vertical rectangle of height 100 ft and width 200ft. Find the total fluid force exerted on the face when the water surface is level with the top of the dam.
21. Write the statement of Pappus Theorem.
22. Find the value of  $\int_0^{\infty} \frac{1}{2}(a^2 - x^2)dx$ .

(8 × 2 = 16 Marks)

SECTION – III

Answer any **six** questions. **Each** question carries **4** marks.

23. Write the Four steps for solving Related Rates Problem.
24. Evaluate  $\lim_{x \rightarrow 0^+} \left( \frac{1}{x} - \frac{1}{\sin x} \right)$ .
25. Find the intervals on which  $f(x) = x^3$  is increasing and the intervals on which it is decreasing.
26. Prove that if  $f$  has an absolute extremum on an open interval  $(a,b)$ , then it must occur at a critical point of  $f$ .
27. Show that the function  $f(x) = \frac{1}{4}x^3 + 1$  satisfies the hypotheses of the Mean Value Theorem over the interval  $[0, 2]$  and find all values of  $c$  in the interval  $(0, 2)$  at which the tangent line to the graph of  $f$  is parallel to the secant line joining the points  $(0, f(0))$  and  $(2, f(2))$ .
28. Find the area of the region that is enclosed between the curves  $y = x^2$  and  $y = x + 6$ .

29. Use cylindrical shells to find the volume of the solid generated when the region  $R$  under  $y = x^2$  over the interval  $[0, 2]$  is revolved about the line  $y = -1$ .
30. Explain the concept of pressure.
31. A plate in the form of an isosceles triangle with base 10ft and altitude 4ft is submerged vertically in machine oil. Find the fluid force  $F$  against the plate surface if the oil has weight density  $\tau = 30 \text{ lb/ft}^3$ .

(6 × 4 = 24 Marks)

#### SECTION – IV

Answer any **two** questions. **Each** question carries **15** marks.

32. Sketch the graph of  $y = (x - 4)^{\frac{2}{3}}$ .
33. A closed cylindrical can is hold 1 liter of liquid. How should we choose the height and radius to minimize the amount of material needed to manufacture the can?
34. Find the area of the surface that is generated by revolving the position of the curve.
- (a)  $y = x^3$  between  $x = 0$  and  $x = 1$  about the  $x$ -axis.
- (b)  $y = x^2$  between  $x = 1$  and  $x = 2$  about the  $y$ -axis.
35. Find the centroid of the region  $R$  enclosed between the curves.
- (a)  $y = x^2$  and  $y = x + 6$ .
- (b)  $y = \sqrt{x}, y = 1, y = 2$  and the  $y$ -axis.

(2 × 15 = 30 Marks)