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Reg. No. : ....

Name : ....

First Semester B.Sc. Degree Examination, January 2024

First Degree Programme under CBCSS

Mathematics

Core Course

MM 1141 : METHODS OF MATHEMATICS

(2023 Admission)

Time : 3 Hours Max. Marks : 80

SECTION I

(All the first **10** questions are compulsory. They carry **1** mark each)

1. When we say f' is increasing on an interval?

2. If $f''(a)$ exists and f has an inflection point at $x = a$, then $f''(a) = ...$

3. Determine whether the statement is true or false: Assume that f is differentiable everywhere. If f is decreasing on [0, 2], then $f(0) > f(1) > f(2)$.

4. If $f'(x_0) = 0$ and $f''(x_0) > 0$, then f has a ——— at x_0.

5. State extreme value theorem.

6. What is meant by a statement?

7. Define contradiction.

8. Are the positive integers well-ordered?

P.T.O.

9. State whether the statement is true or false: Every prime is odd.

10. If $(a,b)=1$ and $(a,c)=1$, then $a,bc)=$

(10 × 1 = 10 Marks)

SECTION II

(Answer any **eight** questions. They question carry **2** marks each)

11. Solve for x: ln x = 3

12. Let $y=\sqrt{x}$. Find formulas for Δy and dy.

13. Show that $f(x)=x^3$ is increasing over the entire x-axis.

14. Let $s(t)=t^3-6t^2$ be the position function of a particle moving along an s-axis, where s is in meters and t in seconds. Find the acceleration function a (t).

15. State the Mean value theorem.

16. Find the critical points of $f(x)=x^3-3x+1$.

17. When we say that a function fig concave up? Give an example.

18. Write an example of a bi-conditional statement.

19. Construct a truth table for the compound statement $(p\vee q)\to(p\wedge q)$.

20. State division algorithm for integers.

21. Find five consecutive integers <100 that are composite numbers.

22. Find the canonical decomposition of 4116.

(8 × 2 = 16 Marks)

SECTION III

(Answer any **six** questions. These question carry **4** marks each)

23. Find all relative extrema of $f(x)=3x^{5/3}-15x^{2/3}$.

24. Find the inflection points, if any, of $f(x)=x^4$.

25. Find the two x-intercepts of the function $f(x) = x^2 - 5x + 4$ and confirm that $f(c) = 0$ at some point c between those intercepts.

26. A garden is to be laid out in a rectangular area and protected by a chicken wire fence. What is the largest possible area of the garden if only 200 running feet of chicken wire is available for the fence?

27. Prove that the number of primes is infinite.

28. Find the number of positive integers ≤ 3000 and divisible by 3, 5, or 7.

29. If a and b are relatively prime, and if $a \mid bc$, prove that $a \mid c$.

30. Write the negation of the following statements

(a) If it is raining then the match is cancelled

(b) Robin is doing his home work and Kiran is practicing his piano lessons

(c) Some drivers do not obey the speed limit

(d) All movies are serious

31. Prove that $\sim (p \vee q) \Leftrightarrow [(\sim p) \wedge (\sim q)]$ using truth tables.

(6 × 4 = 24 Marks)

SECTION – IV

(Answer any **two** questions. These questions carry **15** marks each)

32. (a) The side of a cube is measured with a possible percentage error within ± 2%. Use differentials to estimate the percentage error in the calculated volume of the cube.

(b) Use the first and second derivatives of $f(x) = x^3 - 3x^2 + 1$ to determine the intervals on which f is increasing, decreasing, concave up, concave down. Locate all inflection points.

33. (a) An open box is to be made from a 3ft by 8ft rectangular piece of cardboard by cutting out squares of equal size from the four corners and bending up the sides. Find the maximum volume the box can have?

(b) Show that the function $f(x) = (x^3 / 4) + 1$ satisfies the hypotheses of the Mean-Value Theorem over the closed interval [0, 2] and find all values of c in the interval (0, 2) at which the tangent line to the graph of f is parallel to the secant line joining the points (0, f(0)) and (2, f(2)).

34. (a) Show that every integer $n \geq 2$ has a prime factor.

(b) Using Euclidean algorithm, evaluate (1575, 231)

35. Explain briefly the following proof techniques with suitable examples

(a) Direct proof

(b) Indirect proof

(c) Proof by contradiction

(2 × 15 = 30 Marks)
