S – 2690

Reg. No. : ....

Name : ....

First Semester B.Sc. Degree Examination, January 2024

First Degree Programme under CBCSS

Mathematics

Complementary Course for Physics

MM 1131.1 : Mathematics I — CALCULUS AND SEQUENCE AND SERIES

(2021 Admission onwards)

Time : 3 Hours

Max. Marks : 80

I. All the first **ten** questions are compulsory. They carry **1** mark each.

1. Find $\lim\limits_{x \to 2}(x^2 - x + 1)$.

2. Find x such that $\ln(x + 1) = \sqrt{2}$.

3. State the quotient rule of differentiation.

4. Evaluate $\int \cos^2 x \, dx$.

5. Find $f_x(1, 3)$ for the function $f(x, y) = 2x^3 y^2 + 2y + 4x$.

6. Compute the differential of function $z = xy^2$.

7. State the extreme value theorem for the function of two variables.

P.T.O.

8. Find the limit of the sequence $\dfrac{1}{2}, \dfrac{2}{3}, \dfrac{3}{4}, \dfrac{4}{5}, \dots$

9. Show that $\lim\limits_{n \to +\infty} \sqrt[n]{n} = 1$.

10. Does the series $\sum\limits_{k=1}^{\infty} \dfrac{1}{k}$ converge or diverge?

(10 × 1 = 10 Marks)

II. Answer any **eight** questions. These questions carry **2** marks each.

11. Show that $|x|$ is continuous everywhere.

12. Find $\lim\limits_{x \to -\infty} \dfrac{4x^2 - x}{2x^3 - 5}$.

13. Find the equation of the tangent line to $y = x^2$ at $x = 2$.

14. Find $\int\limits_{1}^{e} \dfrac{1}{x} dx$.

15. Find $\dfrac{d}{dx}\left[\int\limits_{1}^{x} t^3 \, dt\right]$.

16. Find the area of the region under the curve $y = 3\sin x$ over the interval $\left[0, \dfrac{2\pi}{3}\right]$.

17. Find the slope of the surface $z = f(x, y) = x^2 y + 5y^3$ in the x-direction at the point $(1, -2)$.

18. Let $f(x, y) = y^2 e^x + y$. Find f_{xyy}.

19. Let $(x, y) = x^2 e^y$. Find the maximum value of a directional derivative at $(-2, 0)$.

20. Find the limit of the sequence $1, -\dfrac{1}{2}, \dfrac{1}{2^2}, -\dfrac{1}{2^3}, \ldots, (-1)^n \dfrac{1}{2^n}, \ldots$

21. Find the rational number represented by the repeating decimal $0.784784784..$

22. Use the root test to determine whether the series converge or diverge $\displaystyle\sum_{k=2}^{\infty}\left(\dfrac{4k-5}{2k+1}\right)^k$.

<div align="right">(8 × 2 = 16 Marks)</div>

III.   Answer any six questions. These questions carry 4 marks each.

23. Find $\displaystyle\lim_{x\to+\infty}(\sqrt{x^6+5x^3}-x^3)$.

24. Find $\dfrac{d}{dx}[\sin\sqrt{1+\cos x}]$.

25. Evaluate $\displaystyle\int\dfrac{e^x}{\sqrt{1-e^x}}dx$

26. Evaluate $\displaystyle\int_0^1 \tan^{-1}x\,dx$.

27. Find the second-order partial derivatives of $f(x,y)=x^2y^3+x^4y$.

28. Prove that $f(x,y)=x^2+y^2$ is differentiable at (0, 0).

29. Given that $z=e^{xy}$, $x=2u+v$, $y=\dfrac{u}{v}$, find $\dfrac{dz}{du}$ and $\dfrac{\partial z}{\partial v}$ using the chain rule.

30. Test for convergence of series $\displaystyle\sum_{k=1}^{\infty}\dfrac{3k^3-2k^2+4}{k^7-k^3+2}$.

31. Find the nth Maclaurin polynomials for $\cos x$.

<div align="right">(6 × 4 = 24 Marks)</div>

IV. Answer any **two** questions. These questions carry **15** marks each.

32. Let $f(x) = \begin{cases} \dfrac{1}{x+2}, & x < -2 \\ x^2 - 5, & -2 < x \le 3 \\ \sqrt{x+13}, & x > 3 \end{cases}$

   Find

   (a) $\lim\limits_{x \to -2} f(x)$

   (b) $\lim\limits_{x \to 0} f(x)$

   (c) $\lim\limits_{x \to 3} f(x)$.

33. (a) Use implicit differentiation to find $\dfrac{dy}{dx}$ for the Folium of Descartes $x^3 + y^3 = 3xy$.

   (b) Find an equation for the tangent line to the Folium of Descartes at the point $\left(\dfrac{3}{2}, \dfrac{3}{2} \right)$.

   (c) At what point in the first quadrant is the tangent line to the Folium of Descartes horizontal?

34. Locate all relative extrema and saddle points of $f(x,y) = 4xy - x^4 - y^4$.

35. Find the interval of convergence and radius of convergence of the following power series.

   (a) $\sum\limits_{k=0}^{\infty} x^k$

   (b) $\sum\limits_{k=0}^{\infty} k! \, x^k$

   (c) $\sum\limits_{k=0}^{\infty} \dfrac{(-1)^k \, x^k}{3^k (k+1)}$.

   (2 × 15 = 30 Marks)

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