S – 6281

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Reg. No. : ....

Name : ....

First Semester M.Sc. Degree Examination, April 2024

Physics/Physics with Specialization in Nano Science/Physics with Specialization in Space Physics

PH 212/PHNS 512/PHSP 512 : MATHEMATICAL PHYSICS

(2020 Admission Onwards)

Time : 3 Hours

Max. Marks : 75

PART – A

Answer **any five** questions. Each question carries **3** marks.

1. State the properties of the members of a vector space with respect to vector addition and scalar multiplication.
2. Explain the concept of analytic continuation.
3. Obtain the relation between the Fourier transform of a function and its sine and cosine transforms.
4. Given a set of sample measurements $x_i\ i = 1, 2, \cdots, n$ of a random variable X, write down and explain the terms in the expression for the standard deviation of the distribution of X.
5. Distinguish between the three classes of PDEs.
6. State and prove Cayley - Hamilton theorem.
7. Briefly describe kinematics in Riemannian space. Write down the equation of motion for a free particle.
8. Define a Lie group. Give two examples.

(5 × 3 = 15 Marks)

P.T.O.

PART – B

Answer **three** questions. Each question carries **15** marks.

9. (a) Derive the expression for the curl of a vector in general curvilinear co-ordinates.

   (b) Prove the residue theorem.

OR

10. (a) Develop the integral transform of a function $f(x)$ starting from its Fourier series expansion.

    (b) Determine the mean and standard deviation of the outcome of n independent trials of an experiment which has only two outcomes with probability p and $q = (1-p)$ using moment generating function.

11. (a) Develop the expression for the Laplace transform of the n^{th} derivative of a function $f(t)$ in terms of $L[f(t)]$. Illustrate its use for determining $L[f(t)]$ by applying it to the equation $\frac{d^2}{dt^2}\sin kt = -k^2 \sin kt$

    (b) Obtain the series expansion for $J_n(x)$ starting from its generating function. Prove that $J_{-n}(x) = (-1)^n J_n(x)$.

OR

12. (a) Obtain the expression for the covariant derivative of a covariant vector. Hence obtain the expression for the covariant derivative of a contravariant vector.

    (b) Illustrate the use of group theory in particle physics using two instances of its use.

13. (a) Expand $f(x) = x$ in the interval $(0, 2L)$ in a Fourier series.

(b) Prove the addition theorem that if random variables X and Y are normally distributed with the same mean and variance then $Z = X + Y$ has a normal distribution with twice the variance of X and Y.

OR

14. (a) Using partial fraction expansion show that

$$L^{-1}\left\{\frac{l}{(s^2+a^2)(s^2+b^2)}\right\} = \frac{1}{a^2-b^2}\left\{\frac{\sin at}{a} - \frac{\sin bt}{b}\right\} \text{ and}$$

$$L^{-1}\left\{\frac{s^2}{(s^2+a^2)(s^2+b^2)}\right\} = \frac{1}{a^2-b^2}\{a\sin at - b\sin bt\}$$

(b) Given the following Rodrigues formula develop the first four Legendre polynomials.

$$P_n(x) = \frac{1}{2^n n!}\frac{d^n(x^2-1)^n}{dx^n}.$$

(3 × 15 = 45 Marks)

PART – C

Answer any **three** questions. Each question carries **five** marks.

15. Show that complex numbers $(a + ib)$ are isomorphic with 2×2 matrices $\begin{pmatrix} a & b \\ -b & a \end{pmatrix}$

16. Examine the singularities of the function $f(z) = (Z^2 - 1)^{1/2}$ and identify the cut lines possible in the complex plane.

17. Solve the wave equation $\frac{\partial \psi}{\partial t} = a^2 \frac{\partial^2 \psi}{\partial x^2}$ using Fourier series expansion. Hence obtain the dispersion relation.

18. A particle is in simple harmonic motion about $x = 0$. Determine the probability density function $P(x)$ for finding the particle between x and $x + dx$.

19. Develop a series solution for the Hermite differential equation.

$$y'' - 2xy' + 2\alpha y = 0$$

20. Given $T_0 = 1$, $T_1 = x$ and the recurrence relation.

$T_{n+1}(x) - 2x T_n(x) + T_{n-1}(x) = 0$ evaluate $T_2(3)$ and $T_3(2)$

(3 × 5 = 15 Marks)
