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Reg. No. : ....

Name : ....

First Semester M.Sc. Degree Examination, May 2022

Physics

PH 212 : MATHEMATICAL PHYSICS

(2020 Admission onwards)

Time : 3 Hours Max. Marks : 75

PART – A

Answer **any five** questions and each question carries **3** marks.

1. Prove $\left|\vec{u}\times\vec{v}\right| = \left|\vec{u}\right|\left|\vec{v}\right|\sin\theta$.

2. Find the fourier sine transform of e^{-x}.

3. State and prove Cauchy's principle value theorem.

4. Distinguish between the Continuous and discrete variables.

5. Find the Laplace transform of $F(t) = \cosh(kt)$.

6. Define Green's function for a differential operator and explain the reciprocity relation.

7. Prove that, the metric tensor is a fundamental tensor of rank two.

8. Discuss the properties of Special Unitary Group, $SU(n)$.

(5 × 3 = 15 Marks)

P.T.O.

PART – B

Answer **all** the questions and each question carries **15** marks.

9. (a) Using divergence theorem calculate the flux emerging from a vector field $\vec{A} = k\frac{\hat{i}x + \hat{j}y + \hat{k}z}{(x^2 + y^2 + z^2)^{\frac{3}{2}}}$ through surface enclosed by a hemisphere meant by the equations $x^2 + y^2 + z^2 = a^2$ and $z = 0$. **6**

(b) What is residue and derive the general expression for finding the residue of function and evaluate the given integration Evaluate the integral using Cauchy's residue theorem $\int_{-\infty}^{\infty} \frac{dx}{1+x^2}$. **9**

OR

10. (a) Find the Fourier transform of $f(x) = \frac{e^{-ax}}{x}$ and use it to evaluate $\int_0^{\infty} \tan^{-1}\left(\frac{x}{a}\right) \sin x dx$. **7**

(b) Derive an expression for the probability of POISSON DISTRIBUTION? **8**

11. (a) Find the solution of Bessel's differential equation order n is $x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + (x^2 - n^2)y = 0$ by Forbenious method. **9**

(b) Deduce the Rodrigue's Formula for Hermite's Function. **6**

OR

12. (a) Solve, by Green's function method, the initial value problem

$$\ddot{x} + 2\beta\dot{x} + w_0^2 x = F(t)$$

with β positive and small, $x(0) = x_0$, $\dot{x}(0) = v_0$ and $F(t) = \begin{cases} 0, & t < 0 \\ F_{0,} & 0 \le t \le T \\ 0, & t > T \end{cases}$. **7**

(b) Solve the Poission's equation by Green Function method. **8**

13. (a) Deduce the Differential form of a mixed tensor. **9**

(b) Derive an expression for Riemann curvature tensor. **6**

OR

14. (a) Define a group and explain the properties of a group with a set of matrices. **6**

(b) Define the elements of symmetry transformation of a square and find out the representation of matrix elements to the corresponding group. **9**

(3 × 15 = 45 Marks)

PART – C

Answer **any three** questions and each question carries **5** marks.

15. Prove $\nabla \cdot (\vec{r}\, r^{n-1}) = (n+2) r^{n-1}$.

16. State and Prove Chebychev inequality.

17. State and Prove Laplace convolution theorem.

18. Deduce the Recurrence relations in Legendre Function, $l P_l(x) = x P_l'(x) - P_{l-1}'(x)$.

19. Deduce the Ricci Scalar tensor from the Riemann Christoffel tensor.

20. Prove that any finite-dimensional representation of a group of finite order is equivalent to a unitary representation.

(3 × 5 = 15 Marks)