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Reg. No. : ....

Name : ....

First Semester M.Sc. Degree Examination, May 2023

Physics

PH 212 : MATHEMATICAL PHYSICS

(2020 Admission Onwards)

Time : 3 Hours Max. Marks : 75

SECTION – A

Answer any **five** questions. **Each** question carries **3** marks.

1. Find the inverse Laplace transform of $\dfrac{1}{9s^2 + 6s + 1}$

2. What are cyclic groups?

3. Express the square matrix $A = \begin{bmatrix} 1 & 2 & 4 \\ -2 & 5 & 3 \\ -1 & 6 & 3 \end{bmatrix}$ as sum of a symmetric and a skew – symmetric matrix.

4. Prove that the contraction of the tensor A_q^p is a scalar or invariant.

5. Prove the recurrence relation $nP_n = xP_n' - P_{n-1}'$.

6. Determine the analytic function whose real part is $x^3 - 3xy^2 + 3x^2 - 3y^2 + 1$.

P.T.O.

7. The radius r of a cylinder is given as (2.1 ± 0.1) cm and the length l as (6.4 ± 0.2) cm. Find the volume of the cylinder and its standard error.

8. Show that the function $z|z|$ is not analytic anywhere.

(5 × 3 = 15 Marks)

SECTION – B

Answer **all** questions. **Each** question carries **15** marks.

9. (a) Find the Fourier half – range even expansion of the function

$$f(x) = \left(-\frac{x}{l}\right) + 1, \; 0 \le x \le l$$

(b) If the Fourier transform of a function $f(x)$ is $F(s)$, then prove that

$$F[x^n f(x)\} = (-1)^n \frac{d^n}{ds^n} F(s).$$

OR

10. (a) Find the eigen values and normalised eigen vectors of the vectors of the matrix $A = \begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}$.

(b) Find the characteristics equation of the following matrix and verify the Cayley – Hamilton theorem. Given $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 4 \\ 3 & 1 & 1 \end{bmatrix}$.

11. (a) Derive the Rodrigue's formula for Hermite polynomial and hence obtain $H_0(x), H_1(x)$ *and* $H_2(x)$.

(b) Show that $H_n\ (-x) = (-1)^n\ H_n(x)$ and $H_{2n}(0) = (-1)^n \dfrac{(2n)!}{n!}$.

OR

12. (a) Find solution in generalised series from about $x = 0$ of the differential equation $3x\dfrac{d^2y}{dx^2} + 2\dfrac{dy}{dx} + y = 0$ using Frobenius method.

(b) Find the Laplace transform of $\{t^3\delta(t-4)\}$.

13. (a) Prove that $\in_{ilm} = 2\delta_{ij}$ and $\in_{ijk}\in_{ijk} = 6$.

(b) What is Riemann – Christoffel tensor? Discuss its properties.

OR

14. (a) Show that $SU(2)$ group describes rotation and obtain a representation for $SU(2)$ in terms of Pauli matrices.

(b) Show that the permutation elements formed by three objects (1 2 3) form a group.

(3 × 15 = 45 Marks)

SECTION – C

Answer any **three** of the following questions. **Each** question carries **5** marks.

15. Find the Green's function for the boundary value problem $\dfrac{d^2y}{dx^2} - k^2y = f(x)$ with boundary conditions, $y(\pm\infty) = 0$.

16. Find the directional derivative of the scalar point function $\phi = x^2 + xy + x^2$ at the point A(1,–1,–1) in the direction of the line AB where B has coordinates (3,2,1).

17. Find out the conjugate elements and class structure of symmetry group of a square.

18. Using the Laplace transform find the solution of the initial value problem $y'' + 25y = 10 \cos 5t$. Given $y(0) = 2$ and $y'(0) = 0$.

19. Evaluate $\int_{-\infty}^{+\infty} \frac{dx}{(x^2 + 1)^3}$

20. Show that $\vec{F} = (y^2 + 2xz^2)i + (2xy - z)j + (2x^2z - y + 2z)k$ is irrotational and hence find its scalar potential.

(3 $\times$ 5 = 15 Marks)
