(Pages : 4)

T – 1605

Reg. No. : ....

Name : ....

Sixth Semester B.Sc. Degree Examination, April 2024

First Degree Programme under CBCSS

Mathematics

Core Course IX

MM 1641 : REAL ANALYSIS – II

(2018 Admission Onwards)

Time : 3 Hours Max. Marks : 80

SECTION – I

All the first **ten** questions are compulsory. They carry **1** mark each.

1. State composition of continuous functions theorem.
2. State extreme value theorem for continuous functions.
3. Define uniform continuity and give an example.
4. State intermediate value theorem.
5. Find the 10^{th} derivative of $f(x) = x^5 + 4x^2 + 1$.
6. Give an example of a monotone function.
7. When we say that a function is Riemann integrable.
8. Give an example of a set of measure 0.

P.T.O.

9. If $\int_a^b f = 10$, then $\int_b^a f = \cdots$.

10. State Lebesgue's Theorem.

(10 × 1 = 10 Marks)

SECTION – II

Answer any **eight** questions. These questions carry **2** marks each.

11. State sequential criterion for functional limits.

12. Evaluate $\lim_{x \to \pi} (x + \sin x)$.

13. Construct two functions f and g, neither of which is continuous at 0 but $f(x) + g(x)$ is continuous at 0.

14. Whether there exists a continuous function defined on a closed interval with range equal to {1,2,3}.

15. Define Lipschitz Function and give an example of a function which is uniformly continuous but not Lipschitz.

16. Define removable discontinuity with an example.

17. State Darboux's Theorem.

18. State Mean Value Theorem.

19. Find $\lim_{x \to 1} \left(\frac{1-x}{\ln x} \right)$.

20. If P_1 and P_2 are any two partitions of [a, b], then prove that $L(f, P_1) \leq U(f, P_2)$.

21. Distinguish between upper integral and lower integral.

22. If $\int_1^4 f = 4$ and $\int_2^4 f = 1$, then find $\int_1^2 f$.

(8 × 2 = 16 Marks)

SECTION – III

Answer any **six** questions. These questions carry **4** marks each.

23. Using $\varepsilon - \delta$ definition prove that $\lim_{x \to 2}(3x+4) = 10$.

24. Prove that $f(x) = \sqrt{x}$ is uniformly continuous on $[0, \infty)$.

25. Is the converse of intermediate value theorem true? Justify your claim.

26. Let f be differentiable on an open interval (a, b). If f attains a maximum value at some point $c \in (a, b)$ then prove that $f'(c) = 0$.

27. State and prove Rolle's theorem.

28. If $f : A \to R$ is differentiable at a point $c \in A$. then prove that f is continuous at c. Is the converse true? Justify your answer.

29. If $g : A \to R$ is differentiable on an interval A and satisfies $g'(x) = 0$ for all $x \in A$, then prove that $g(x) = k$ for some constant $k \in R$.

30. Assume that $f_n \to f$ uniformly on $[a, b]$ and that each f_n is integrable. Prove that f is integrable and $\lim_{n \to \infty} \int_a^b f_n = \int_a^b f$.

31. Prove that the Dirichlet's function $g(x) = \begin{cases} 1 \text{ for } x \text{ rational} \\ 0 \text{ for } x \text{ irrational} \end{cases}$ is not integrable.

(6 × 4 = 24 Marks)

SECTION – IV

Answer any **two** questions. These questions carry **15** marks each.

32. Let $f : A \to R$ be continuous on A. If $K \subseteq A$ is compact, then prove that $f(K)$ is also compact.

33. State and prove chain rule for derivatives.

34. (a) Prove that a bounded function f is integrable on $[a, b]$ if and only if, for every $\varepsilon > 0$, there exists a partition P_ε of $[a, b]$ such that $U(f, P_\varepsilon) - L(f, P_\varepsilon) < \varepsilon$.

    (b) Prove that if f is continuous on $[a, b]$, then it is integrable.

35. State and prove the fundamental theorem of integral calculus.

(2 × 15 = 30 Marks)
