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Reg. No. : ..............................

Name : ...................................

**Sixth Semester B.Sc. Degree Examination, April 2024**

**First Degree Programme under CBCSS**

**Mathematics**

**Core Course XII**

**MM 1644 : LINEAR ALGEBRA**

**(2021 Admission)**

Time : 3 Hours Max. Marks : 80

SECTION – A

All the first **ten** questions are compulsory. Each question carries **1** mark.

1. Find the intersection of the lines $x + y = 3$ and $x - y = 1$.

2. For which values of $\alpha$, is there a whole line of solution for the following equation

$$\alpha x + 4y = 0$$
$$4x + 4\alpha y = 0$$

3. If $A = \begin{bmatrix} 1 & 2 & 3 \\ -1 & 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$ then express $AB$ as the linear combination of columns of $A$.

4. Define Subspace of a vector space over a field $F$.

P.T.O.

5. Find a basis for the vector space $\mathbb{R}^3$ over $\mathbb{R}$.

6. Find the null space of the matrix $A = \begin{bmatrix} 1 & -2 \\ -2 & 4 \end{bmatrix}$.

7. Define the rank of a matrix.

8. Find the determinant of the matrix $\begin{bmatrix} -1 & 2 & 1 \\ 2 & -4 & -2 \\ 1 & 1 & 7 \end{bmatrix}$.

9. Find the Eigen values of $\begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$.

10. What is the relation between trace of a matrix and its Eigen values?

**(10 × 1 = 10 Marks)**

SECTION – B

Answer any **eight** questions. Each question carries **2** marks.

11. Find the values of $\beta$ such that the system,

$$2x - 6y = \beta$$
$$-x + 3y = -4$$

(a) Has no solution (b) Has infinite solution

12. Find two points on the line of intersection of the three planes $t = 0$ and $z = 0$ and $x + y + z + t = 1$ in four-dimensional space.

13. What are the elementary transformation in order to make the system

$x + 2y = 3$

$3x + 5y = 8$, an upper triangular system. Also write the transformed system and find the solution.

14. Is matrix multiplication commutative? Justify.

15. Let $v_1, v_2, \ldots v_n$ are linearly independent vectors in a vector space $V$. Then, what is the dimension of the span of these vectors? If we add a vector $v_{n+1}$, where $v_{n+1}$ is a linear combination of $v_1, v_2, \ldots \ldots, v_n$, then what the change in the dimension of the span of these $(n+1)$ vectors?

16. Write down the 2 by 2 matrices $A$ and $B$ that have entries $a_{ij} = i + j$ and $b_{ij} = (-1)^{i+j}$ multiply them to find $AB$ and $|AB|$.

17. Check whether the set $W = \{(x, y) \in \mathbb{R}^2 ; x + y = 1\}$ is a subspace of $\mathbb{R}^2$ or not?

18. Show that the columns of the matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ are linearly independent.

19. Using Crammers Rule solve $x_1 + 3x_2 = 0,\ 2x_1 + 4x_2 = 6$.

20. Draw the triangle with vertices $A = (0,0), B = (2,2)$, and $C = (-1,3)$. Find its area.

21. Find the Eigen values of the matrix $A$ and $A^2$ if $A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$.

22. Give examples of $A$ and $B$ such that, $A + B$ is not invertible although $A$ and $B$ are invertible.

**(8 × 2 = 16 Marks)**

## SECTION – C

Answer any **six** questions. Each question carries **4** marks.

23. Apply Elimination and back substitution to solve

$$x - 2y + z = 0;\ \ 2y - 8z = 8;\ -4x + 5y + 9z = -9$$

24. Prove that the eigenvalues of a triangular matrix are the entries on its main diagonal.

25. Find $L$ and $U$ such that $LU = A$; where $A = \begin{bmatrix} 1 & 2 \\ 3 & 8 \end{bmatrix}$.

26. The matrix that rotates the $x - y$ plane by an angle $\theta$ is $A(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$.

(a) Verify $A(\theta_1)A(\theta_2) = A(\theta_1 + \theta_2)$

(b) Find $A(\theta)A(-\theta)$

27. Find the reduced echelon form of $A = \begin{bmatrix} 1 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 1 \end{bmatrix}$. What is the rank?

28. Find the dimension and basis of subspaces

(a) column space and

(b) row space – of the matrix $A = \begin{bmatrix} 0 & 3 & 3 & 3 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$

29. Find the Eigen values of matrix $\begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$.

30. Let $u = [1, -2, -5]^T$, $v = [2, 5, 6]^T$ and $w = [7, 4, -3]^T$. Verify whether $\{u, v, w\}$ is independent or not if it is not independent, express $w$ as the linear combination of $u$ and $v$.

31. Let $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ be the linear map such that $T(1, 0) = (2, 3), T(0, 1) = (1, 4)$. Find $T$.

**(6 × 4 = 24 Marks)**

## SECTION – D

Answer any **two** questions. Each question carries **15** marks.

32. (a) Under what condition on $b_1, b_2, b_3$ is the following system solvable? Include $b$ as a fourth column in $[A\, b]$. Find all solutions when that condition holds :

$$x + 2y - 2z = b_1$$
$$2x + 5y - 4z = b_2$$
$$4x + 9y - 8z = b_3$$

(b) What conditions on $b_1, b_2, b_3, b_4$ make the following system solvable? Find $x$ in that case.

$$\begin{bmatrix} 1 & 2 \\ 2 & 4 \\ 2 & 5 \\ 3 & 9 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix}$$

33. Diagonalize the following matrix $A = \begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$.

34. Reduce $A$ to $U$ and find det $A$ = product of the pivots if

(a) $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 2 & 3 \end{bmatrix}$

(b) $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 2 & 3 \\ 3 & 3 & 3 \end{bmatrix}$

35. (a) Show that $S = \{(1,1,1),(1,1,0),(1,0,0)\}$ form a basis for $\mathbb{R}^3$.

(b) Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be defined by $T(x, y, z) = (2y + z, x - 4y, 3x)$

(i) Show that $T$ is Linear

(ii) Find the null space of $T$ (Kernel of $T$)

**(2 × 15 = 30 Marks)**

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