(Pages : 4) **T – 1607**

Reg. No. : ....

Name : ....

Sixth Semester B.Sc. Degree Examination, April 2024

First Degree Programme under CBCSS

Mathematics

Core Course X

MM 1642 : COMPLEX ANALYSIS – II

(2021 Admission)

Time : 3 Hours Max. Marks : 80

SECTION – I

Answer **all** questions.

1. State Morera's theorem.

2. State generalized Cauchy's integral formula.

3. Evaluate $\int\limits_{|z|=4} \frac{1}{z-2}\, dz$.

4. Define uniform convergence in sequence.

5. Find $\sum_{j=0}^{\infty}\left(\frac{1}{3}\right)^{j}$.

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6. Using the ratio test, show that $\sum_{j=0}^{\infty} \frac{j^2}{4^j}$ converges

7. Find the Maclaurin's series for sinz.

8. Find the singularities of $f(z) = \frac{\cos z}{z^2 (z-\pi)^3}$.

9. Define pole. Give an example.

10. Find the poles of $f(z) = \frac{z^2}{z^2 + 4}$.

(10 × 1 = 10 Marks)

SECTION – II

Answer any **eight** questions.

11. Compute $\int_{|z|=1} \frac{e^{5z}}{z^3} \, dz$.

12. Show that $\int_{|z|=3} \frac{e^z}{z-2} \, dz = 2\pi i e^2$.

13. Find $\int_C \frac{dz}{z-1}$ where C is the circle $|z| = 3$.

14. Show that $1 + c + c^2 + ... = \frac{1}{1-c}$, if $|c| < 1$.

15. If $\sum_{j=0}^{\infty} c_j$ sums to S and λ is any complex number then show that $\sum_{j=0}^{\infty} \lambda c_j$ sums to λS.

16. Prove that $\lim_{n\to\infty} (n!)^{\frac{1}{n}} = \infty$.

17. Expand $e^{\frac{1}{z}}$ in a Laurent series around z = 0.

18. Find the residue of *f*(*z*) tanz at $z = \frac{\pi}{2}$.

19. Find the residue at $z = 0$ of $f(z) = \frac{5z-2}{z(z-1)}$.

20. Determine the order of each pole and the value of residue there for $f(z) = \frac{1-e^{2z}}{z^4}$.

21. Prove that $\lim_{n\to\infty} (n!)^{\frac{1}{n}} = \infty$.

22. Find the Maclaurin series expansion of sinhz.

(8 × 2 = 16 Marks)

SECTION – III

Answer any **six** questions.

23. Find $\int_C \frac{e^z + \sin z}{z} dz$ where C is the circle $|z-2| = 3$.

24. If f is analytic in a domain D, show that all its derivatives f', f''..... exist and are analytic in D.

25. Evaluate $\int_{|z|=3} \frac{z^2+5}{(z-2)^2} dz$.

26. State and prove ratio test.

27. Find the first five terms of the Maclaurin's series for tanz.

28. If R is the radius of convergence of $\sum a_n z^n$ then what is the redii of convergence of $\sum a_n^2 z_n$ and $\sum a_n z^{2n}$.

29. Compute the residue at singularity of $f(z) = \dfrac{\cos z}{z^2(z-\pi)^3}$.

30. Find PV $\int_{-\infty}^{\infty} \dfrac{x \sin x}{(1+x^2)} dx$.

31. Evaluate $\int_{|z-1|=1} \dfrac{2z^2+z}{z^2+1} dz$ using Cauchy Residue theorem.

(6 × 4 = 24 Marks)

SECTION – IV

Answer any **two** questions.

32. State and prove Cauchy's integral formula.

33. (a) State Picard's theorem and verify it for $e^{\frac{1}{z}}$ near $z=0$.

    (b) Explain zeroes and different types of singularities with examples.

34. (a) State and prove Cauchy Residue theorem.

    (b) Using Cauchy Residue theorem, evaluate $\oint_{|z|=2} \dfrac{1-2z}{z(z-1)(z-3)} dz$.

35. Evaluate $\int_0^{\pi} \dfrac{d\theta}{2-\cos\theta}$.

(2 × 15 = 30 Marks)