(Pages : 4) **T – 1608**

Reg. No. : ....

Name : ....

Sixth Semester B.Sc. Degree Examination, April 2024

First Degree Programme under CBCSS

Mathematics

MM 1643 : ABSTRACT ALGEBRA : RING THEORY

(2018 Admission Onwards)

Time : 3 Hours Max. Marks : 80

SECTION – I

Answer **all** the first ten questions. Each carries **1** mark.

1. Give a proper nontrivial subring of Z_8.

2. Give an example for an integral domain.

3. Let A be a subring of ring R. If $r \in R$, $a \in A$ implies $ra \in A$ then A is called __________.

4. Give an example for a ring R and subring of it that is not an ideal.

5. Define ring homomorphism.

6. Which is true: Z homomorphic to Z_n or Z isomorphic to Z_n?

7. State the division algorithm for $F[x]$ where F is a field.

8. Define associates in an integral domain.

P.T.O.

9. Define Euclidean Domain.

10. In what type of integral domain, irreducibles and primes are the same?

(10 × 1 = 10 Marks)

SECTION – II

Answer any **eight** questions. Each carries **2** marks.

11. Show that in a ring R, if $a, b \in R$, then $a.0 = 0$ and $a(-b) = (-ab)$.

12. Show that left (multiplicative) cancellation holds in an integral domain.

13. What is meant by an ideal generated by $a_1, a_2, ...a_n$ in a ring R? Find ideal generated by x^2 in $Z[x]$.

14. If A and B are ideals in a ring R, show that $A + B$ is an ideal.

15. If R is a commutative ring with characteristic 2, show that $a \to a^2$ is a homomorphism on R.

16. If rings R, S are isomorphic, show that $R[x]$ and $S[x]$ are isomorphic.

17. Is $a + ib \to |a + ib|$ a homomorphism from the set of all complex numbers C to C? Justify.

18. Is $x^2 + 1$ irreducible over Z_3? Justify.

19. Define a unique factorization domain. Give an example.

20. Show that if F is a field, then $F[x]$ is a Euclidean domain.

21. On an integral domain D, define $a \sim b$ if a and b are associates. Show that this is an equivalence relation on D.

22. Give two factorizations of 21 in $Z[\sqrt{-5}]$.

(8 × 2 = 16 Marks)

SECTION – III

Answer any **six** questions. Each carries **4** marks.

23. Show that every nonzero element of Z_n is either a unit or a zero divisor. Is this true for Z?

24. Show that if R is a ring with unity, then R/A is an integral domain if and only if A is a prime ideal.

25. Let $R = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} : a, b, c, d \in Z \right\}$ and let I be a subset of R with even entries at all the places in its elements. Show that it is an ideal of R. Find out the elements in $\frac{R}{I}$.

26. Show that $f(x) = 5x$ is a ring homomorphism from Z_4 to Z_{10}.

27. If D is an integral domain, prove that D [x] is also an integral domain.

28. Show that the product of two primitive polynomials is primitive.

29. Show that every Euclidean domain is a PID.

30. In a PID, show that every strictly increasing chain of ideals must be finite in length.

31. Show that the integral domain $Z\left[\sqrt{-5}\right]$ is not a UFD.

(6 × 4 = 24 Marks)

SECTION – IV

Answer any **two** questions. Each question carries **15** marks.

32. (a) If R is a commutative ring with unity, A an ideal in it, show that $\frac{R}{A}$ is a field if and only if A is maximal.

(b) Show that $< x >$ is a prime ideal in $Z[x]$, but not maximal.

33. (a) Describe the subrings of the ring of all integers.

(b) Which of these a prime ideals? Why?

(c) Let $a \in R$, a ring. Is $S = \{x \in R : ax = 0\}$ a subring? Is this an ideal? Justify your answers.

34. State and prove the theorem on unique factorization of nonzero, non unit elements in $Z[x]$.

35. Prove that every PID is a UFD.

(2 × 15 = 30 Marks)
