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S – 3417

Reg. No. : ....

Name : ....

Third Semester B.Sc. Degree Examination, February 2024

First Degree Programme under CBCSS

Mathematics

Core Course

MM 1341 : ELEMENTARY NUMBER THEORY AND CALCULUS I

(2018 Admission onwards)

Time : 3 Hours

Max. Marks : 80

SECTION – I

All the questions are compulsory. **Each** question carries **1** mark

1. State the Pigeonhole Principle.

2. State the Fundamental Theorem for Arithmetic.

3. Define continuity of a vector valued function.

4. If $r_0 = r(t_0), v_0 = r'(t_0)$, give the vector equation of the tangent line to the graph of $r(t)$ at r_0.

5. Define Unit Tangent Vector.

6. Give the formula for a_T, the tangential component of acceleration for a moving particle in terms of the velocity v and the acceleration a.

P.T.O.

7. Find the natural domain of $f(x,y)=\sqrt{y+1}+\ln(x^2-y)$.

8. Define boundary point of a set.

9. Define directional derivative of $f(x,y,z)$ in the direction u.

10. Give the equation of the tangent plane to a level surface S at a point (x_0, y_0, z_0).

(10 × 1 = 10 Marks)

SECTION – II

Answer any **eight** questions. **Each** question carries **2** marks

11. Prove that there are infinitely many primes.

12. Express 3014 in base 8.

13. Prove that if p is a prime and $p \backslash ab$, then $p \backslash a$ or $p \backslash b$.

14. Give the general solution of an LDE $ax+by=c$, if (x_0, y_0) is its particular solution.

15. Find the parametric equations that represent a line in 3-space that passes through the point (1,0,0) and is parallel to the vector (–1,3,2).

16. Find the derivative of $r(t)=t^2 i+e^t j-(2\cos \pi t)k$.

17. If $r_1(t)$ and $r_2(t)$ are two vector functions of t, derive the expression for $\frac{d}{dt}(r_1 \cdot r_2)$.

18. A particle moves along a circular path in such a way that its x – and y – coordinates at time t are $x=2\cos t, y=2\sin t$. Find the instantaneous velocity and speed of the particle at time t.

19. Define Level Surfaces of $f(x,y,z)$ and find the level surfaces of $f(x,y,z)=x^2+y^2+z^2$.

20. For $f(x,y)=x^2y+5y^3$, find the slope of the surface $z = f(x, y)$ in the x-direction in the y-direction at the point (1, –2).

21. Let $f(x,y)=x^2e^y$. Find the maximum value of a directional derivative at (–2,0).

22. State the Second Partial test for finding the relative extrema of a function.

(8 × 2 = 16 Marks)

SECTION – III

Answer any **six** questions. **Each** question carries **4** marks.

23. Let a and b are any positive integers. Prove that the number of positive integers $\leq a$ and divisible by b is $[a \backslash b]$.

24. Prove that every integer $n \geq 2$ has a prime factor.

25. Prove that two integers a and b are relatively prime if and only if there are integers α and β such the $\alpha a+\beta b=1$.

26. Prove that, if $r(t)$ is a vector-valued function and $r(t)$ is differentiable at t, then $r't=x'(t)i+y'(t)j$. Also for vector-valued functions $r_1(t)$ and $r_2(t)$, Prove that $\frac{d}{dt}[r_1(t)+r_2(t)]=\frac{d}{dt}[r_1(t)]+\frac{d}{dt}[r_2(t)]$.

27. If $r(t)$ is a differentiable vector-valued function in 2-space or 3-space and $\|r(t)\|$ is constant for all t, then $r(t)\cdot r'(t)=0$.

28. Give the formula for arc Length L from $t = a$ to $t = b$, for the graph of a smooth vector-valued function $r(t)$. Also Find the arc length of that portion of the circular helix, $x=\cos t, y=\sin t, z=t$ from $t=0$ to $t=\pi$.

29. Assuming that polynomials in one variable and sine function are continuous, show that $f(x, y) = \sin(3x^2y^5)$ is continuous everywhere State the results used for the proof.

30. Prove that if a function $f(x,y)$ is differentiable at a point, then it is continuous at that point.

31. Suppose that $w=\sqrt{x^2+y^2+z^2}, x=\cos\theta, y\sin\theta, z=\tan\theta$. Use the chain rule to find $\frac{dw}{d\theta}$ when $\theta=\frac{\pi}{4}$.

(6 × 4 = 24 Marks)

SECTION – IV

Answer any **two** questions. **Each** question carries **15** marks.

32. (a) State and prove the Division Algorithm for integers.

    (b) Using Euclidean Algorithm express (4076, 1024) as a linear combination of 4076 and 1024.

33. (a) Find $T(t)$ and $N(t)$ for the circular helix $x = a \cos t$ t, $y = a \sin t$, $z = ct$ where $a > 0$.

    (b) Find the curvature for the ellipse $r = 2\cos t\, i + 3 \sin t\, j (0 \leq t \leq 2\pi)$.

34. Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{dy}$ if $z = x^4 \sin(xy^3)$. Given $f(x,y) = -\frac{xy}{x^2+y^2}$, Find the limit of $f(x,y)$ as $(x,y) \to (0,0)$ limit along.

    (a) the x–axis

    (b) the y–axis

    (c) the line $y = x$

    (d) the line $y = -x$

    (e) the parabola $y = x^2$

35. Explain Lagrange's Multiplier method for finding extreme values and find the points on the sphere $x^2 + y^2 + z^2 = 36$ that are closest to and farthest from the point (1,2,2).

(2 × 15 = 30 Marks)

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