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T – 2533

Reg. No. : ....

Name : ....

Fourth Semester B.Sc. Degree Examination, July 2024

First Degree Programme under CBCSS

Mathematics

Complementary Course for Physics

MM 1431.1 : MATHEMATICS IV- FOURIER SERIES, COMPLEX ANALYSIS AND PROBABILITY THEORY

(2021 Admission Onwards)

Time : 3 Hours

Max. Marks : 80

PART – A

Answer **all** questions. Each question carries **1** mark.

1. State Dirichlet conditions for the convergence of a fourier series of a function $f(x)$ of period 2π.

2. State linearity property of Fourier Transform.

3. State the formula for the Fourier transform of $f(x)$.

4. State De Moivre's theorem.

5. Find the residue of $\frac{\sin z}{z}$ at $z = 0$.

6. Define residue.

7. Find the real part of $\frac{1}{z}$.

P.T.O.

8. A number selected from numbers 1 to 10 at random. What is the probability of choosing an odd number?

9. Find the Binomial distribution whose mean is 9 and variance is 6.

10. Define Sample space.

(10 × 1 = 10 Marks)

PART – B

Answer any **eight** questions. Each question carries **2** marks.

11. Write down the Euler formula for calculating the Fourier coefficient.

12. Find the Fourier series of the function $f(x) = x, -\pi < x < \pi$.

13. Find the Fourier Cosine transform of $f(x) = e^{-ax}$, where $a > 0$.

14. If $F(s)$ is the complex Fourier transform of $f(x)$, then show that $F(f(x-a)) = e^{isa} F(s)$.

15. Calculate the residue of $\dfrac{z+1}{z^2 - 2z}$ at its poles.

16. Distinguish between pole and essential singular points.

17. Let $f(z) = z^2 + 3z$. Find U and V and Calculate the value of f at $z = 1 + 4i$.

18. State Cauchy's Integral formula.

19. What is the probability of showing two heads and one tail. When three coins are tossed?

20. Given $P(A) = \frac{1}{4}, P(B) = \frac{1}{3}$ and $P(A \cup B) = \frac{1}{3}$, evaluate $P(A/B)$ and $P(A \cap B')$.

21. What is the chance that a leap year selected at random will contain 53 Sundays?

22. A letter of English alphabet is chosen at random. What is the probability that it is one of the letter in the word probability.

(8 × 2 = 16 Marks)

PART – C

Answer any **six** questions. Each question carries **4** marks.

23. Find the Fourier series of the function $f(x)=\begin{cases}0 & if \quad -2<x<-1 \\ k & if \quad -1<x<1 \\ 0 & if \quad 1<x<2\end{cases}$.

24. Prove that $U = 2x - x^3 + 3xy^2$ is harmonic and find its harmonic conjugate.

25. If 3% of electric bulbs manufactured by a company are defective. Find the probability that in a sample of 100 bulbs exactly five bulbs are defective.

26. Evaluate $\int_C \frac{4z-3}{z(z-1)} dz$. Where C is the circle $|z| = 2$.

27. A random variable x has the following probability function

Values of x :	−2	−1	0	1	2	3
$p(x)$:	0.1	k	0.2	$2k$	0.3	k

Find the value of k and calculate $P(X<0)$ and $P(X>1)$.

28. Find the Fourier transform of $f(x)$ given by

$$f(x)=\begin{cases}1-x^2 & if |x|<1 \\ 0 & \text{otherwise}\end{cases}$$

29. Expand $f(z)=\frac{1}{(z+1)(z+3)}$ in Laurent's series valid for $1<|z|<3$.

30. Suppose two dice are thrown. If X denote the sum of the numbers on the dice, find the probability density function of X.

31. Find the Fourier sine transform of $e^{-|x|}$.

(6 × 4 = 24 Marks)

PART – D

Answer any **two** questions. Each question carries **15** marks.

32. (a) Find the Fourier transform of e^{-ax^2} where $a>0$.

(b) Find the Fourier transform of $f(x)=\begin{cases} a^2-x^2 & \text{for } |x|\le a \\ 0 & \text{for } |x|>a \end{cases}$

Hence deduce that $\int_0^{\infty}\frac{\sin t - t\cos t}{t^3}dt=\frac{\pi}{4}$.

33. (a) Evaluate $\int_C \frac{\sin \pi z^2+\cos \pi z^2}{(z-1)^2(z-2)}dz$ where C is $|z|=3$.

(b) Prove that the function $f(z)=e^x(\cos y - i\sin y)$ is nowhere differentiable.

34. (a) Evaluate $\int_0^{2\pi}\frac{1}{5+3\cos\theta}d\theta$.

(b) X is a normal variate with mean 30 and S.D 5, find the probabilities that

(i) $26\le X\le 40$,

(ii) $X\ge 45$ and

(iii) $|X-30|>5$.

35. (a) In a normal distribution, 31% of the items are under 45 and 8% are over 64. Find the mean and S.D of the distribution.

(b) Out of 800 families with 5 children each, how many would you expect to have

(i) 3 boys,

(ii) 5 girls and

(iii) Either 2 or 3 boys?

Assume equal probabilities for boys and girls.

(2 × 15 = 30 Marks)