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Reg. No. : ....

Name : ....

Fourth Semester B.Sc. Degree Examination, July 2024

First Degree Programme under CBCSS

Mathematics

MM 1441 : ELEMENTARY NUMBER THEORY AND CALCULUS – II

(2018 Admission Onwards)

Time : 3 Hours

Max. Marks : 80

SECTION – A

Answer **all** questions. Each question carries **1** mark.

1. Find the remainder when 16^{53} is divided by 7.

2. Determine whether the linear congruence $16x \equiv 18 \pmod{12}$ is solvable.

3. Find an inverse of 5 modulo 7.

4. Determine whether $N = 54, 893, 534, 046$ is a square.

5. State chinese Remainder theorem.

6. Evaluate $\int_0^1 \int_{x^2}^{x} dA$.

7. Find the limits of integration $\iint_R (3x - 2y)\, dA$ where R is the region bounded by the circle $x^2 + y^2 = 1$.

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8. Write the converting formula for 3-dimensional Cartesian to spherical coordinates.

9. Show that $\phi(x,y,z) = x^2 - 3y^2 + 4z^2$ is a potential function for the vector field $\bar{F}(x,y,z) = 2x\hat{i} - 6y\hat{j} + 8z\hat{k}$.

10. State the fundamental theorem of line integrals.

(10 × 1 = 10 Marks)

SECTION – B

Answer any **eight** questions. Each question carries **2** marks.

11. If $a \equiv b \pmod{m}$ and $c \equiv d \pmod{m}$, then show that $(a - c) \equiv (b - d) \pmod{m}$.

12. Solve the congruence $49x \equiv 84 \pmod{35}$.

13. Using casting out nines, check if the sum of the numbers 3569, 24387 and 49508 is 78464.

14. Show that a positive integer a is self invertible modulo p if and only if $a \equiv \pm 1 \pmod{p}$.

15. Evaluate $\int_1^2 \int_0^4 2xy\, dy\, dx$.

16. Use a double integral to find the volume of the solid that is bounded above by the plane $z = 4 - x - y$ and below by the rectangle $R = [0, 1] \times [0, 2]$.

17. Evaluate $\iint_R e^{-(x^2+y^2)}\, dA$ where R is the region enclosed by the circle $x^2 + y^2 = 1$.

18. Find the surface area of that portion of paraboloid $z = x^2 + y^2$ below the plane $z = 1$.

19. Evaluate the triple integral $\iiint_G 12xy^2 z^3\, dV$ over the rectangular box G defined by the inequalities $-1 \le x \le 2, 0 \le y \le 3, 0 \le z \le 2$.

20. Find the curl of the vector field $\vec{F}(x,y,z) = x^2 y\hat{i} + 2y^3 z\hat{j} + 3z\hat{k}$.

21. Evaluate $\int_C (xy + z^3)\, ds$ from (1, 0, 0) to (–1, 0, π) along the helix C given by $x = \cos t$, $y = \sin t$, $z = t$ $(0 \le t \le \pi)$.

22. Show that the vector field $\vec{F}(x,y) = 2xy^3\hat{i} + (1+3x^2y^2)\hat{j}$ is a conservative field on the entire plane.

(8 × 2 = 16 Marks)

SECTION – C

Answer any **six** questions. Each question carries **4** marks.

23. Prove that $a \equiv b \pmod{m}$ if and only if a and b leave the same remainder when divided by m.

24. Prove that no integer of the form $8n + 7$ can be expressed as a sum of three squares.

25. Using the Pollard rho method, factor the integer 8051.

26. If p is a prime, then prove that $(p-1)! \equiv -1 \pmod{p}$.

27. Calculate $\iint_R \frac{\sin x}{x}\, dA$ where R is the triangle in the xy-plane bounded by the x-axis, the line $y = x$, and the line $x = 1$.

28. Change the order of integration and hence evaluate $\int_0^2 \int_{x^2}^{2x} (4x+2)\, dy\, dx$.

29. Evaluate $\iint_R \frac{x-y}{x+y}\, dA$ where R is the region enclosed by the lines $x - y = 0$, $x - y = 1$, and $x + y = 3$.

30. Find the work done by the force field $\vec{F}(x,y,z) = xy\hat{i} + yz\hat{j} + xz\hat{k}$ on a particle that moves along the curve $C : \vec{r}(t) = t\hat{i} + t^2\hat{j} + t^3\hat{k}$ $(0 \le t \le 1)$.

31. Using Green's theorem, evaluate the integral $\oint_C (x^2 - y^2)\,dx + x\,dy$, where C is the circle $x^2 + y^2 = 9$.

(6 × 4 = 24 Marks)

SECTION – D

Answer any **two** questions. Each question carries **15** marks.

32. (a) Using Chinese Remainder theorem solve the linear system of congruence $x \equiv 3 \pmod 7$; $x \equiv 3 \pmod 5$ and $x \equiv 4 \pmod{12}$. **7**

(b) Find the primes p for which $\frac{2^{p-1} - 1}{p}$ is a square. **8**

33. (a) Use a triple integral of find the volume of the solid within the cylinder $x^2 + y^2 = 9$ and between the planes $z = 1$ and $x + z = 5$. **7**

(b) Use spherical coordinates to find the volume of the solid that is within the sphere $x^2 + y^2 + z^2 = 9$, outside the cone $z = \sqrt{x^2 + y^2}$, and above the xy– plane. **8**

34. Verify the divergence theorem for $\vec{F}(x,y,z) = 2x\hat{i} - yz\hat{j} + z^2\hat{k}$ where the surface σ is the paraboloid $z = x^2 + y^2$ capped by the disk $x^2 + y^2 \le 1$ in the plane $z = 1$.

35. Verify Stoke's theorem for the vector field $\vec{F}(x,y,z) = (z - y)\hat{i} + (z + x)\hat{j} - (x + y)\hat{k}$, taking σ to be the portion of the paraboloid $z = 9 - x^2 - y^2$ above the xy– plane with upward orientation.

(2 × 15 = 30 Marks)
