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Reg. No. : ...............................

Name : ...................................

# First Semester M.Sc. Degree Examination, February 2025

## Physics/Physics with Specialization in Nano Science/Physics with Specialization in Space Physics

## PH 212/PHNS 512/PHSP 512 : MATHEMATICAL/PHYSICS

(2020 Admission Onwards)

Time : 3 Hours　　　　　　　　　　　　　　　　　　　　Max. Marks : 75

### PART – A

Answer any **five** questions. Each question carries **3** marks.

1. Verify whether eigen values of the matrix $\begin{pmatrix} 1 & 2 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$ are in the $disc|\lambda - 1| \leq 2$.

2. When will the matrices $\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$ and $\begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}$ commutes under multiplication. Verify.

3. What is the projection of a vector $\vec{a} = 2\hat{x} - 3\hat{y} + 6\hat{z}$ on vector $\vec{b} = \hat{x} - 3\hat{y} + 5\hat{z}$ ?

4. If $f(z) = (x^2 + ay^2) + ibxy$ is a complex analytic function of $z = x + iy$, what are the values of $a$ and $b$.

5. What is the Laplace transform $f(t) = t^2$ ?

P.T.O.

6. Solve the differential equation $\frac{dy}{dx} = \frac{y}{x}$.

7. Why a set of matrices under multiplication do not make Abelian group.

8. Define contravariant and covariant tensors with examples.

(5 × 3 = 15 Marks)

PART – B

Answer three questions each question carries 15 marks.

9. (a) Given the eigen values $\lambda_1 = 1$, $\lambda_2 = -1$ and corresponding eigen values $f_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$, $g_1 = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$, $f_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ and $g_2 = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix}$ Construct $A$ and verify $Af_n = \lambda_n g_n$ and $Ag_n = \lambda_n f_n$.

(b) In the spherical polar coordinate system, $q_1 = r, q_2 = \theta, q_3 = \varphi$. The transformation equations are $x = r\sin\theta\cos\varphi,\ y = r\sin\theta\sin\varphi, z = r\cos\theta$. Calculate the spherical polar coordinate scale factors: $(h_r, h_\theta, h_\varphi)$.

OR

10. (a) Obtain the Fourier series of a triangular wave.

(b) Explain different distributions in probability.

11. (a) Find the Laplace transform of

(i) Dirac delta function.

(ii) $\sin(\omega_0 t)$

(iii) $\cos(\omega_0 t)$

(b) Consider the differential equation $x\frac{d^2y}{dx^2} + (1-2n)\frac{dy}{dx} + xy = 0$. Check whether $x^n J_n(x)$ is a solution of the differential equation.

OR

12. (a) Show that $(1-2xt-t^2)^{-1/2}$ is a generating function of $P_n(x)$.

(b) Prove that following recurrence relation.

(i) $nP_n(x)=(2n-1)xP_n(x)-(n-1)P_{n-1}(x)$

(ii) $nP_n(x)=xP_n(x)-P_{n-1}(x)$

13. (a) In a 3D system, find values of

(i) $\delta_{ii}$

(ii) $\delta_{ij}\varepsilon_{ijk}$

(b) What is $C_{3v}$, group? Obtain the group multiplication table for the same.

OR

14. (a) What do you mean by character table? Write down character table for $C_{4v}$ group.

(b) Define covariant tensor, if $dS^2=g_{ij}dx^i dx^j$ is invariant, show that $g_{ij}$ is a symmetric covariant tensor of rank 2.

(3 × 15 = 45 Marks)

PART – C

Answer any **three** questions. Each question carries **5** marks.

15. With any vector $\vec{A}, \vec{A}.\nabla\vec{r}=A$ verify the result in Cartesian and polar coordinates.

16. State and prove Cauchy's integral formula.

17. Fit a straight line using principle of least square.

18. Find the value of $J_{n-1}(x)+J_1(x)$.

19. Define a group, Construct a group of order 4 and justify that the group satisfy all properties of a group.

20. Obtain the metric tensor for two dimensional plane in polar coordinates.

(3 × 5 = 15 Marks)