(Pages : 4)

T – 2548

Reg. No. : .....

Name : .....

Fourth Semester B.Sc. Degree Examination, July 2024

First Degree Programme under CBCSS

Mathematics

Complimentary Course for Chemistry / Polymer Chemistry

MM 1431.2 : MATHEMATICS IV – DIFFERENTIAL EQUATIONS, VECTOR CALCULUS AND ABSTRACT ALGEBRA

(2018-2020 Admission)

Time : 3 Hours

Max. Marks : 80

SECTION – I

All the first **ten** questions are compulsory. They carry **1** mark each.

1. Solve $\frac{dy}{dx} = 2xe^{x^2}$.
2. Find the Wronskian of $y_1 = \sin x$ and $y_2 = \cos x$.
3. Write the Laplace transform of $\sin \omega t$.
4. Find the integrating factor of $y' + \frac{y}{x} = x^2$.
5. If $f(x,y) = xy$, then find $\nabla f(x,y)$ at (2, 4).
6. Define conservative vector field.
7. State Gauss Divergence Theorem.
8. Define the identity element of a Group $(G,*)$.
9. Write the property of a group $(G,*)$ to be an abelian group.
10. Write an example for non-abelian Group.

(10 × 1 = 10 Marks)

P.T.O.

SECTION – II

Answer any **eight** questions. These question carries **2** each.

11. Solve $\frac{dy}{dx}+\frac{x}{y}=0, y(0)=2$.

12. Find the general solution of $y''-8y'+15y=0$.

13. Solve $x^2y''-5xy'+9y=0$.

14. Find the Particular integral of $y''+y=x^2+1$.

15. Evaluate the line integral $I=\int_c F.dr$, where $F=(x+y)i+(y-x)j$, along the parabola $y^2=x$ from (1, 1) to (4, 2).

16. Use Green's theorem to find the area of the circle $x^2+y^2=a^2$.

17. Find the vector area of the surface of the hemisphere $x^2+y^2+z^2=a^2$ with $z\geq 0$.

18. Show that $\varphi(x,y)=\tan^{-1}xy$ is the scalar potential function of $F=\frac{1}{1+x^2y^2}(y\,i+x\,j)$.

19. Prove that two cosets are either disjoint or identical.

20. Define the centre of a Group G.

21. Define a homomorphism from a group G to G'.

22. Let $\alpha=\begin{bmatrix}1&2&3&4&5&6\\2&1&3&5&4&6\end{bmatrix}$ and $\beta=\begin{bmatrix}1&2&3&4&5&6\\6&1&2&4&3&5\end{bmatrix}$. Find $\beta\alpha$ and $\alpha\beta$.

(8 × 2 = 16 Marks)

SECTION – III

Answer any **six** questions. These question carries **4** marks each.

23. Verify the equation $x \log x \frac{dy}{dx} + y = 2 \log x$ is linear and hence solve.

24. Solve $y'' - 5y' + 6y = \sin x + 10 \cos x$.

25. Show that the vector field $F = 3x^2yz^2i + \left(x^3 z^2 + e^z\right)j + \left(2x^3yz + ye^z\right)k$ is conservative and find its potential function.

26. Evaluate the surface integral $I = \int_S a.ds$ where $a = xi$ and S is the surface of the hemisphere $x^2 + y^2 + z^2 = a^2$ with ≥ 0.

27. Find the volume enclosed between a sphere of radius a centred at origin and a circular cone of a half angle α with its vertex at origin.

28. Define a group with an example.

29. Define

    (a) Odd permutation

    (b) Even permutation

    (c) Alternating group A_n.

30. Define quaternion group Q. Determine the conjugacy classes of Q.

31. Consider the group $G = (\mathbb{Z}_4, +)$ and $G' = (\mathbb{Z}_4, +)$. Let $\phi : G \to G'$ defined by $\phi(n) = n \bmod (4)$

    (a) Find ker (ϕ).

    (b) Show that ker (ϕ) is a subgroup of G.

(6 × 4 = 24 Marks)

SECTION – IV

Answer any **two** questions. These question carries **15** marks each.

32. Solving the following

(a) $2y\frac{dy}{dx} = e^{x-y^2}$, $y(4) = -2$.

(b) $(1+y^2)\frac{dx}{dy} + x = e^{\tan^{-1}y}$.

(c) $(\cos x - 2xy)dx + (e^y - x^2)\,dy = 0$; $y(0) = 0$.

33. Using Method of Variation of Parameter solve.

(a) $\frac{d^2y}{dx^2} + 4y = \tan 2x$.

(b) $\frac{d^2y}{dx^2} + y = x \sin x$.

34. (a) State Green's theorem in plane.

(b) Verify Greens theorem in plane $\int_C (3x^2 - 5y^2)\,dx + (4y - 6xy)\,dy$ where C is the boundary for the region enclosed by the parabola $y = x^2$ and $y^2 = x$.

35. (a) Find the right and left cosets of the subgroup $H = \{0,2,4\}$ in the group G = {0, 1, 2, 3, 4, 5} under addition modulo 6.

(b) Sate Lagrange's Theorem in Group theory.

(c) Suppose G is a group with order $|G| = 24$. Let H be a subgroup of G with order $H = 6$. Using Lagrange's theorem in group theory, answer the following questions:

(i) How many left cosets of H are there in G?

(ii) How many distinct right cosets of H are there in G?

(iii) What is the index of H in G.

(2 × 15 = 30 Marks)
