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Reg. No. : ....

Name : ....

Fourth Semester B.Sc. Degree Examination, July 2024

First Degree Programme under CBCSS

Mathematics

Complementary Course for Chemistry/Polymer Chemistry

MM 1431.2 : MATHEMATICS IV – DIFFERENTIAL EQUATIONS, VECTOR CALCULUS AND ABSTRACT ALGEBRA

(2021 Admission Onwards)

Time : 3 Hours Max. Marks : 80

SECTION – A

Answer **all** questions. Each carries **1** mark.

1. Write the order and degree of the differential equation $\frac{d^2y}{dx^2} + 3\left(\frac{dy}{dx}\right)^3 + y = 0$

2. Find an integrating factor for the differential equation $y \log y dx + (x - \log y)dy = 0$

3. Write the general form of a linear differential equation of the n^{th} order.

4. Find the Wronskian of $\sin 2x$ and $\cos 2x$.

5. Define a conservative vector field.

P.T.O.

6. If C is the unit circle $x^2 + y^2 = 1$, oriented counter clockwise and $\overline{F}(x,y) = x\hat{i} + y\hat{j}$, then $\int_C \overline{F}.d\overline{r} =$ ______

7. State Gauss Divergence theorem.

8. Find all solutions of $x + 15^7 = 3$ in z_{15}.

9. Find the order of the subgroup of z_4 generated by 3.

10. Describe all units in the ring z.

(10 × 1 = 10 Marks)

SECTION – B

Answer any **eight** questions. Each carries **2** marks.

11. Find the differential equation whose set of independent solution is $\{e^x, xe^x\}$.

12. Check whether the differential equation $ye^{xy}dx + (xe^{xy} + 2y)dy = 0$ is exact.

13. Solve $(D^3 - 4D^2 + 4D)y = 0$.

14. Transform the differential equation $x^2 \frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = \log x$ to a linear differential equation with constant coefficients.

15. Find the curl of the vector $\overline{F} = x^2\hat{i} - 3\hat{j} + yz^2\hat{k}$.

16. Evaluate $\int_C (3x^2 + y^2)dx + 2xydy$ along the circular arc C given by $x = \cos t$, $y = \sin t$, $0 \le t \le \pi/2$.

17. Find $\iint_\sigma (x + y + z)ds$ where σ is the triangular region with vertices (1,0,0), (0,1,0) and (0,0,1).

18. Using divergence theorem, find the outward flux of the vector field $\overline{F}(x,y,z) = 2x\hat{i} + 3y\hat{j} + z^2\hat{k}$ across the unit cube $0 \le x \le 1$, $0 \le y \le 1$, $0 \le z \le 1$.

19. Determine whether the binary operation * defined on z^+ by $a * b = 2^{ab}$ is associative.

20. Write the group table of the Klein 4-group.

21. Compute $z\sigma$ if $z = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 4 & 1 & 3 & 6 & 5 \end{pmatrix}$ and $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 1 & 4 & 5 & 6 & 2 \end{pmatrix}$.

22. Differentiate unit and unity in a ring.

(8 × 2 = 16 Marks)

SECTION – C

Answer any **six** questions. Each carries **4** marks.

23. Solve: $xdy - ydx = \sqrt{x^2 + y^2}\,dx$.

24. Solve $x^2p^2 + xyp - 6y^2 = 0$.

25. Solve by the method of variation of parameters $\frac{d^2y}{dx^2} - y = \frac{2}{1+e^x}$.

26. Prove: $div(\varphi\overline{F}) = \varphi div\overline{F} + \nabla\varphi.\overline{F}$.

27. Find a potential function φ for the field $\overline{F}(x,y) = 2xy^3\hat{i} + (1+3x^2y^2)\hat{j}$.

28. Using Green's theorem, evaluate $\int_C (y - \sin x)dx + \cos x dy$ where C is the plane triangle enclosed by the lines $y = 0$, $x = \frac{\pi}{2}$, $y = \frac{2}{\pi}x$.

29. Show that the binary structure $(R,+)$ is isomorphic to $(R^+,\cdot)$ where + is usual addition and $\cdot$ is usual multiplication.

30. Find all subgroups of z_{18} and draw its subgroup diagram.

31. Show that $a^2 - b^2 = (a+b)(a-b)$, $\forall a, b \in R$, where R is a ring if and only if R is commutative.

(6 × 4 = 24 Marks)

SECTION – D

Answer any **two** questions. Each carries **15** marks.

32. (a) Solve: $(3y - 7x + 7)dx + (7y - 3x + 3)dy = 0$

(b) Solve $(x^2 y - 2xy^2)dx - (x^3 - 3x^2 y)dy = 0$

33. (a) Find the orthogonal trajectories of the family of confocal conics $\frac{x^2}{a^2} + \frac{y^2}{a^2 + \lambda} = 1$, where λ is the parameter.

(b) Solve $\frac{d^2 y}{dx^2} - 4y = x \sinh x$.

34. Verify Stoke's theorem for $\overline{F} = (x^2 + y^2)\hat{i} - 2xy\hat{j}$ taken around the rectangle bounded by the lines $x = \pm a$, $y = 0$, $y = b$.

35. (a) Show that $(Q^+, *)$ is an abelian group, where $*$ is defined by $a * b = \frac{ab}{3}$, $a, b \in Q^+$.

(b) Let R be a ring end let a be a fixed element of R. Let $I_a = \{x \in R \mid ax = 0\}$. Show that I_a is a subring of R.

(2 × 15 = 30 Marks)