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Reg. No. : .....

Name : .....

Second Semester B.Sc. Degree Examination, August 2024

First Degree Programme under CBCSS

Mathematics

Foundation Course II

MM 1221 : FOUNDATIONS OF MATHEMATICS

(2023 Admission)

Time : 3 Hours Max. Marks : 80

SECTION – I

Answer **all** questions. Each question carries **1** mark. Answer in **one** word to maximum of **two** sentences.

1. Suppose that a particle moves on a coordinate line so that its velocity at time t is $v(t) = t^2 - 2t \ m/s$. Find the displacement of the particle during the time interval $0 \le t \le 3$.

2. Show that if a body released from rest (initial velocity zero) is in free fall, then its average velocity over a time interval $[0, T]$ during its fall is its velocity at time $t = \frac{T}{2}$.

3. Evaluate $\int \frac{t^2 - 2t^4}{t^4} dt$.

4. Find the circumference of a circle of radius a from the parametric equations $x = a \cos t, y = a \sin t, 0 \le t \le 2\pi$.

P.T.O.

5. Find the rectangular coordinates of the point P whose polar coordinates are $(r, \theta) = \left(6, \frac{2\pi}{3}\right)$.

6. Find the equation of the hyperbola with vertices $(0, \pm 8)$ and asymptotes $y = \pm\frac{4}{3}x$.

7. Find the distance between the points (1, –2, 0) and (4, 0, 5).

8. Find the norms of $v = (-2, 3)$ and $w = (2,3,6)$.

9. Calculate the scalar triple product $u.(v \times w)$ of the vectors $u = 3i - 2j - 5k$, $v = i + 4j - 4k$, $w = 3j + 2k$.

10. Find parametric equation of the line passing through (1, 2, –3) and parallel to $v = 4i + 5j - 7k$.

(10 × 1 = 10 Marks)

SECTION – II

Answer any **eight** questions. They carry **two** marks each.

11. Suppose that a curve $y = f(x)$ in the xy-plane has the property that at each point (x, y) on the curve, the tangent line has slope x^2. Find an equation for the curve given that it passes through the point (2, 1).

12. Evaluate $\int \frac{e^{\sqrt{x}}}{\sqrt{x}} dx$.

13. Evaluate $\sum_{k=1}^{30} k(k+1)$.

14. Find the average value of the function $f(x) = \sqrt{x}$ over the interval [1, 4], and find all points in the interval at which the value of f is the same as the average.

15. Sketch the graphs of $r = 1$ in polar coordinates.

16. Find an equation of the parabola that is symmetric about the y-axis, has its vertex at the origin, and passes through the point (5, 2).

17. Find the new coordinates of the point (–2, 6) if the coordinate axes are rotated through an angle of $\theta = 60°$.

18. Find the slope of the tangent line to the polar curve $r = 2 \sin \theta$ for $\theta = \frac{\pi}{6}$.

19. Find the center and radius of the sphere $x^2 + y^2 + z^2 - 2x - 4y + 8z + 17 = 0$.

20. Find the angle that the vector $v = -\sqrt{3}i + j$ makes with the positive x-axis.

21. Find the direction cosines of the vector $v = 2i - 4j + 4k$ and approximate the direction angles to the nearest degree.

22. Find a vector that is orthogonal to both of the vectors u = (2, –1, 3) and v = (–7, 2, –1).

(8 × 2 = 16 Marks)

SECTION – III

Answer any **six** questions. They carry **4** marks each.

23. Use Newton's Method to approximate the real solutions of $x^3 - x - 1 = 0$.

24. Evaluate $\int \cos^3 x \, dx$.

25. Find the total area between the curve $y = 1 - x^2$ and the x-axis over the interval [0, 2].

26. Graph the parametric curve $x = 2t - 3$, $y = 6t - 7$ by eliminating the parameter, and indicate the orientation on the graph.

27. Find the area of the region enclosed by the rose curve $r = \cos 2\theta$.

28. Suppose that the axes of an xy-coordinate system are rotated through an angle of $\theta = 45°$ to obtain an $x'y'$-coordinate system. Find the equation of the curve $x^2 - xy + y^2 - 6 = 0$ in $x'y'$-coordinates.

29. If u and v are nonzero vectors in 2-space or 3-space, and if θ is the angle between them, then show that $\cos \theta = \frac{u, v}{\|u\| \|v\|}$.

30. Find an equation for the line L of intersection of the planes $2x-4y+4z=6$ and $6x+2y-3z=4$.

31. Describe the surface $z=(x-1)^2+(y+2)^2+3$.

(6 × 4 = 24 Marks)

SECTION – IV

Answer any **two** questions. They carry **15** marks each.

32. (a) A penny is released from rest near the top of the Empire State Building at a point that is 1250 ft above the ground. Assuming that the free-fall model applies, how long does it take for the penny to hit the ground, and what is its speed at the time of impact.

(b) Find the average value of $f(x)=\sec x \tan x$ over the interval $\left[0, \frac{\pi}{3}\right]$.

33. Sketch the graph of $r^2=4\cos 2\theta$ in polar coordinates.

34. (a) Find the points on the cardioid $r=1-\cos\theta$ at which there is a horizontal tangent line, a vertical tangent line, or a singular point.

(b) Find the distance D between the point (1, –4, –3) and the plane $2x-3y+6z=-1$.

35. (a) Find parametric equations of the line L passing through the points $P_1(2,4,-1)$ and $P_2(5,0,7)$. Where does the line intersect the xy-plane?

(b) Find the spherical coordinates of the point that has rectangular coordinates $(x,y,z)=(4,-4,4\sqrt{6})$.

(2 × 15 = 30 Marks)