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Reg. No. : .....

Name : .....

Second Semester B.Sc. Degree Examination, August 2024

First Degree Programme under CBCSS

Statistics

Complementary Course for Mathematics

ST 1231.1 – PROBABILITY AND RANDOM VARIABLES

(2022 Admission Onwards)

Time : 3 Hours Max. Marks : 80

SECTION – A

Answer **all** questions. Each question carries **1** mark.

1. What do you mean by random experiments?
2. Define simple events and composite events.
3. Give the frequency definition of probability.
4. Prove that $P(\emptyset)=0$.
5. Define conditional probability.
6. Define compound probability.
7. How do you get the marginal probability density function from the joint probability density function of a vector of continuous random variables?.

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8. Express variance in terms of conditional variance and conditional expectation.

9. State the linearity property of expectation.

10. Why do you say that the characteristic function of a random variable always exists?

(10 × 1 = 10 Marks)

SECTION – B

Answer any **eight** questions. Each question carries **2** marks.

11. Distinguish between mutually exclusive and exhaustive events with examples.

12. If $A \subset B$, then prove that $P(A) \leq P(B)$.

13. Give the classical definition of probability and mention its draw backs.

14. Define the terms "a posteriori" probability and "priori" probability.

15. If $P(A)=0.3$, $P(B)=0.6$, $P(A \cup B)=0.7$, find $P(B/A)$.

16. Define the probability density function of a continuous random variable X and mention its properties.

17. If $f(x/y)=\frac{cx}{y^2}$, $0 \leq x < y \leq 1$ is a conditional pdf, find c?

18. If X has a pmf $f(x)=\frac{1}{3}$, $x=1,2,3$ and 0 elsewhere. Find the pmf of $Y=2X+1$.

19. If X and Y are any two random variables write $V(aX \pm bY)$, where a and b are real constants.

20. Write the expression for the rth and sth product moment about the origin of the bivariate random vector (X,Y).

21. Show that $0_x(t)$ and $0_x(-t)$ are complex conjugate functions, where $0_x(t)$ is the characteristic function of the random variable X.

22. State the Cauchy-Schwartz inequality.

(8 × 2 = 16 Marks)

SECTION – C

Answer any **six** questions. Each question carries **4** marks.

23. For any two events A and B in the same sample space prove that

$$P[(A\cap B^c)\cup(A^c\cap B)]=P(A)+P(B)-2P(A\cap B).$$

24. Give the axiomatic definition of probability.

25. State and prove the law of total probability.

26. Five men are selected at random on 5 chairs around a table. What is the probability that a particular pair to be seated adjacently?

27. Explain the mistakes if any in the following statements (i) $P(A)=0.1$, $P(A\cap B)=0.2$, (ii) $P(A)=0.87$, $P(A\cup B)=0.78$.

28. Find the conditional pdf of X/Y and that of Y/X, if the joint pdf of (X,Y) is $f(x,y)=3xy$, $0<x,y<1$.

29. If X has the pdf $f(x)=\frac{1}{2}$, $-1<x<1$ and 0 elsewhere, what is the pdf of $Y=X^2$?

30. Two unbiased dice are thrown. Find the expectation of the sum of the number of points on them.

31. A continuous r.v X has the pdf $f(x)=2x$, $0<x\le 1$, and 0 elsewhere. Find (a) $F(x)$, (b) $P\left(X\le\frac{1}{2}\right)$ and (c) $P\left(\frac{1}{4}\le X\le\frac{3}{4}\right)$.

(6 × 4 = 24 Marks)

SECTION – D

Answer any **two** questions. Each question carries **15** marks.

32. The news papers A, B and C are published in a city. It is estimated from a survey that 20% of the adults read A, 16% read B, 14% read C, 8% read both A and B, 5% read both A and C, 4% read both B and C while 2% read all the three. What percentage of the adults read at least one of the news papers?

33. (a) State and prove the Baye's Theorem.

    (b) In a bolt factory, machines M_1, M_2 and M_3 manufacture respectively 25,35 and 40 percent of the total output. Of their outputs 5, 4 and 2 percent respectively are defective bolts. One bolt taken at random from the product and is found to be defective. What is the probability that it is manufactured by machine M_2?

34. For the joint pdf $f(x,y)=x+y$, $0<x<1$, $0<y<1$, obtain the marginal pdfs of X and Y and also $P\left(X>\frac{1}{2}/Y>\frac{1}{2}\right)$ and check whether X and Y are independent or not.

35. Let the joint pdf of a discrete random vector (X,Y) be given in the table.

X \ Y	0	1	2	3
0	0	1/8	1/4	1/8
1	1/8	1/4	1/8	0

Find $E(X/Y=2)$, $E(Y/X=1)$, $V(X/Y=2)$ and $V(Y/X=1)$.

(2 × 15 = 30 Marks)