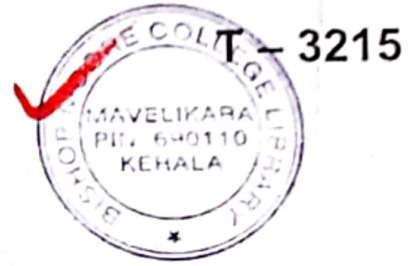


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Reg. No. :

Name :

Second Semester B.Sc. Degree Examination, August 2024

First Degree Programme under CBCSS

Mathematics

Complementary Course for Chemistry/Polymer Chemistry

MM 1231.2 : MATHEMATICS II -

INTEGRAL CALCULUS AND VECTOR DIFFERENTIATION

(2021 Admission onwards)

Time : 3 Hours

Max. Marks : 80

SECTION - I

Answer all questions. They carry 1 mark each.

1. The iterated integral $\int_0^1 \int_1^2 f(x, y) dy dx$ integrates f over the rectangle defined by $0 < x < 1$, $1 < y < 2$.
2. The integral $\int e^{3x} dx = \frac{1}{3} e^{3x} + C$, if $u = 3x$.
3. Evaluate $\int x \sin 3x dx$.
4. Find the points of intersection of the circle $x^2 + y^2 = 4$ and $y = x + 2$.
5. The surface area of the surface of revolution that is generated by revolving the portion of the curve $y = f(x)$ between $x = a$ and $x = b$ about x - axis is _____.
6. Find the rectangular coordinate of the point whose polar coordinate $\left(4, \frac{\pi}{3}\right)$.

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7. The double integral formula for finding the area of the region R is _____.
8. Find $r'(t)$, if $r(t) = 4i - \cos t j$.
9. The parametric equation $x = \cos t$, $y = \sin t$ represent a _____.
10. Find the distance between the points $(0, 1)$ and $(1, 2)$.

(10 × 1 = 10 Marks)

SECTION – II

Answer any **eight** questions. These question carry **2** marks each.

11. Evaluate $\int_1^9 \sqrt{x} dx$.
12. Evaluate $\int_0^1 (2x + 1)^3 dx$.
13. Evaluate $\int_0^1 x e^x dx$.
14. Find the area between the curves $y = x^2$ and $y = \sqrt{x}$ over the interval $\left[\frac{1}{4}, 1\right]$.
15. Find the arc length of the curve $y = x^2$ over the interval $[0, 1]$.
16. Find the Maclaurin series of $f(x) = \tan^{-1} x$.
17. Evaluate $\int_2^4 \int_0^1 x^2 y dx dy$.
18. Evaluate $\int_0^1 \int_{-x}^{x^2} y^2 x dy dx$.
19. Evaluate triple integral $\int_{-1}^1 \int_0^2 \int_0^1 dx dy dz$.
20. Find the parametric equation corresponding to the vector equation $r(t) = ti + t^3 j + k$.
21. Show that $\vec{r}(t) = 3 \cos t i + 4 \sin t j + k$ is continuous at $t = 0$.
22. Write the vector equation of the line segment joining the points $(1, 2)$ and $(4, 6)$.

(8 × 2 = 16 Marks)

SECTION – III

Answer any **six** questions. These questions carry **4** marks each.

23. Find the surface area of that portion of the surface $z = \sqrt{4 - x^2}$ that lies above the rectangle R in the xy plane whose coordinates satisfy $0 \leq x \leq 1$ and $0 \leq y \leq 4$.
24. Find the directional derivative of $f(x, y) = e^{xy}$ at $(-2, 0)$ in the direction of the unit vector that makes an angle $\frac{\pi}{3}$ with a positive x -axis.
25. Evaluate $\iint_R xy \, dA$ over the region R Enclosed between $y = \frac{x}{2}$, $y = \sqrt{x}$, $x = 2$, $x = 4$.
26. Find the parametric equations of the line tangent to the graph $r(t) = e^{2t}i + (2 - \ln t)j$ at $t = 1$.
27. Evaluate $\int \cos^4 x \, dx$.
28. Evaluate $\int 2 \tan x \, dx$.
29. Evaluate $\int \frac{dx}{\sqrt{2 - x^2}}$.
30. Find the area of the surface that is generated by revolving the portion of the curve $y = x^3$ between $x = 0$ and $x = 1$ about the x -axis.
31. Find the maximum value of the directional derivative of $f(x, y) = \ln(x^2 + y^2)$ at $(1, 1)$ and find the unit vector where the maximum value occurs.

(6 × 4 = 24 Marks)

SECTION – IV

Answer any **two** questions. These questions carry **15** marks each.

32. (a) Use polar coordinates to evaluate $\int_{-1}^1 \int_0^{\sqrt{1-x^2}} (x^2 + y^2)^{\frac{3}{2}} dy dx$.
- (b) Use triple integration in cylindrical coordinates to find the volume of the solid G is bounded above by the hemisphere $z = \sqrt{25 - x^2 - y^2}$ below by the xy plane and laterally by the cylinder $x^2 + y^2 = 9$.
33. (a) Show that the graphs of $r_1(t) = t^2i + tj + t^3k$ and $r_2(t) = (t-1)i + \frac{t^2}{4}j + (5-t)k$ intersect at point $(1,1,3)$. Find the degree measure of acute angle between the tangent lines to the graph of $r_1(t)$ and $r_2(t)$ at the point $(1,1,3)$.
- (b) A heat-seeking particle is located at the point $P(1,4)$ on a flat metal plate whose the temperature at a point (x,y) is $T(x,y) = 5 - 4x^2 - y^2$. Find the parametric equations for the trajectory of the particle if it moves continuously in the direction of maximum temperature increase.
34. (a) Evaluate the integral $\int \frac{2x+4}{x^3-2x^2} dx$.
- (b) Find the area of the region enclosed by the graphs $y = \frac{1}{\sqrt{1-9x^2}}$ above x -axis on the Interval $\left[0, \frac{1}{6}\right]$.
35. (a) Derive the formula for the volume of a right pyramid whose altitude is h and whose base is a square with sides of length a .
- (b) Use cylindrical shells to find the volume of the solid generated when the region R in the first quadrant enclosed between $y = x$ and $y = x^2$ is revolved about the y axis.

(2 × 15 = 30 Marks)