(Pages : 4) **T – 3215**

Reg. No. : ....

Name : ....

Second Semester B.Sc. Degree Examination, August 2024

First Degree Programme under CBCSS

Mathematics

Complementary Course for Chemistry/Polymer Chemistry

MM 1231.2 : MATHEMATICS II –

INTEGRAL CALCULUS AND VECTOR DIFFERENTIATION

(2021 Admission onwards)

Time : 3 Hours Max. Marks : 80

SECTION – I

Answer **all** questions. They carry **1** mark each.

1. The iterated integral $\int_0^1\int_1^2 f(x, y)\, dydx$ integrates f over the rectangle defined by $-- < x < —, -- < y < ---$.

2. The integral $\int e^{3x} dx = f --- du$, if $u = 3x$.

3. Evaluate $\int x \sin 3x\, dx$.

4. Find the points of intersection of the circle $x^2 + y^2 = 4$ and $y = x + 2$.

5. The surface area of the surface of revolution that is generated by revolving the portion of the curve $y = f(x)$ between $x = a$ and $x = b$ about x - axis is ____________.

6. Find the rectangular coordinate of the point whose polar coordinate $\left(4, \frac{\pi}{3}\right)$.

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7. The double integral formula for finding the area of the region R is ________.

8. Find $r'(t)$, if $r(t) = 4i - \cos t\ j$.

9. The parametric equation $x = \cos t,\ y = \sin t$ represent a ________.

10. Fine the distance between the points $(0, 1)$ and $(1, 2)$.

(10 × 1 = 10 Marks)

SECTION – II

Answer any **eight** questions. These question carry **2** marks each.

11. Evaluate $\int_1^9 \sqrt{x}\, dx$.

12. Evaluate $\int_0^1 (2x+1)^3 dx$.

13. Evaluate $\int_0^1 xe^x\ dx$.

14. Find the area between the curves $y = x^2$ and $y = \sqrt{x}$ over the interval $\left[\frac{1}{4} 1\right]$.

15. Fine the arc length of the curve $y = x^2$ over the interval *[0 1]*.

16. Fine the Maclaurin series of $f(x) = \tan^{-1} x$.

17. Evaluate $\int_2^4 \int_0^1 x^2 y\ dx\ dy$.

18. Evaluate $\int_0^1 \int_{-x}^{x^2} y^2 x dy dx$.

19. Evaluate triple integral $\int_{-1}^1 \int_0^2 \int_0^1 dx dy dz$.

20. Find the parametric equation corresponding to the vector equation $r(t) = ti + t^3 j + k$.

21. Show that $\vec{r}\ (t) = 3\cos t\ i + 4\ \sin t\ j + k$ is continuous at $t = 0$.

22. Write the vector equation of the line segment joining the points $(1, 2)$ and $(4,6)$.

(8 × 2 = 16 Marks)

SECTION – III

Answer any **six** questions. These questions carry **4** marks each.

23. Find the surface area of that portion of the surface $z = \sqrt{4 - x^2}$ that lies above the rectangle R in the xy plane whose coordinates satisfy $0 \le x \le 1$ and $0 \le y \le 4$.

24. Find the directional derivative of $f(x, y) = e^{xy}$ at $(-2, 0)$ in the direction of the unit vector that makes an angle $\frac{\pi}{3}$ with a positive x -axis.

25. Evaluate $\iint_R xy\, dA$ over the region R Enclosed between $y = \frac{x}{2}$, $y = \sqrt{x}$, $x = 2$, $x = 4$.

26. Find the parametric equations of the line tangent to the graph $\vec{r}(t) = e^{2t} i + (2 - \ln t) j$ at $t = 1$.

27. Evaluate $\int \cos^4 x\, dx$.

28. Evaluate $\int 2 \tan x\, dx$.

29. Evaluate $\int \frac{dx}{\sqrt{2 - x^2}}$.

30. Find the area of the surface that is generated by revolving the portion of the curve $y = x^3$ between $x = 0$ and $x = 1$ about the $x - axis$.

31. Find the maximum value of the directional derivative of $f(x, y) = ln(x^2 + y^2)$ at $(1,1)$ and find the unit vector where the maximum value occurs.

(6 × 4 = 24 Marks)

SECTION – IV

Answer any **two** questions. These questions carry **15** marks each.

32. (a) Use polar coordinates to evaluate $\int_{-1}^{1}\int_{0}^{\sqrt{1-x^2}} (x^2+y^2)^{\frac{3}{2}}\ dydx$.

(b) Use triple integration in cylindrical coordinates to find the volume of the solid G is bounded above by the hemisphere $z=\sqrt{25-x^2-y^2}$ below by the xy plane and laterally by the cylinder $x^2+y^2=9$.

33. (a) Show that the graphs of $r_1(t)=t^2i+tj+t^3k$ and $r_2(t)=(t-1)i+\frac{t^2}{4}j+(5-t)K$ intersect at point (1,1,3). Find the degree measure of acute angle between the tangent lines to the graph *of* $r_1(t)$ and $r_2(t)$ at the point (1,1,3).

(b) A heat-seeking particle is located at the point P(1,4) on a flat metal plate whose the temperature at a point (x,y) is $T(x,y)=5-4x^2-y^2$. Find the parametric equations for the trajectory of the particle if it moves continuously in the direction of maximum temperature increase.

34. (a) Evaluate the integral $\int \frac{2x+4}{x^3-2x^2}dx$.

(b) Find the area of the region enclosed by the graphs $y=\frac{1}{\sqrt{1-9x^2}}$ above x -axis on the Interval $\left[0, \frac{1}{6}\right]$.

35. (a) Derive the formula for the volume of a right pyramid whose altitude is h and whose base is a square with sides of length a.

(b) Use cylindrical shells to find the volume of the solid generated when the region R in the first quadrant enclosed between $y=x$ and $y=x^2$ is revolved about the y axis.

(2 × 15 = 30 Marks)