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Reg. No. : ....

Name : ....

First Semester B.Sc. Degree Examination, March 2023

First Degree Programme Under CBCSS

Mathematics

Complementary Course I for Chemistry and Polymer Chemistry

MM 1131.2 : MATHEMATICS I – CALCULUS WITH APPLICATIONS IN CHEMISTRY I

(2018–2020 Admission)

Time : 3 Hours Max. Marks : 80

SECTION – A

Answer **all** the questions. **Each** question carries **1** mark.

1. Find the derivative of $f(x) = x^3 \sin x$.

2. Find $\frac{dy}{dx}$ if $y = \ln\left(a^x + a^{-x}\right)$.

3. Multiply the complex numbers $1+2i$ and $3-4i$.

4. Define $\sinh x$.

5. By De Moivre's theorem, $(\cos\theta + \vec{i}\sin\theta)^n = \ldots$

6. Define the scalar product of two vectors.

7. Identify the surface $|\vec{r}| = k$.

P.T.O.

8. Find the unit vector in the direction of the vector $\vec{i} + \vec{j}$.

9. Evaluate the integral $I = \int \ln x dx$.

10. Evaluate $I = \int \frac{1}{\sqrt{1-x^2}} dx$.

(10 × 1 = 10 Marks)

SECTION – B

Answer **any eight** questions. **Each** question carries **2** marks.

11. Verify Rolles's Theorem for the function $f(x) = \sin x$ on $[0, 2\pi]$.

12. Find $\frac{dw}{dt}$, if $w = \tan x$ and $x = 4t^3 + t$.

13. Using mean value theorem determine inequalities satisfied by sin x for suitable ranges of the real variable x.

14. Find the modulus and argument of the complex number $z = 2 - 3i$.

15. Find the complex conjugate of the complex number $z = w^{3y+2tx}$, where $w = x + 5i$.

16. Express z in the form $x + iy$, when $z = \frac{3 - 2\vec{i}}{-1 + +4\vec{i}}$.

17. Find the area A of the parallelogram with sides $\vec{a} = \vec{i} + 2\vec{j} + 3\vec{k}$ and $b = 4\vec{i} + 5\vec{j} + 6\vec{k}$.

18. Find the direction of the line of intersection of the two planes $x + 3y - z = 5$ and $2x - 2y + 4z = 3$.

19. Find the angle between the vectors $\vec{a} = \vec{i} + 2\vec{j} + 3\vec{k}$ and $b = 2\vec{i} + 3\vec{j} + 4\vec{k}$.

20. Evaluate the integral $I = \int_0^\infty \frac{x}{x^2 + a^{2^2}} dx$.

21. Find the mean value of the function $f(x) = x^2 - 1$ on $[0, \sqrt{3}]$.

22. Evaluate the integral $I = \int x^2 \sin x dx$.

(8 × 2 = 16 Marks)

SECTION – C

Answer **any six** questions. **Each** question carries **4** marks.

23. Find the magnitude of the radius of curvature at a point (x, y) on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

24. Find the positions and nature of the stationary points of the function $f(x) = x^3 - 3x^2 + 3x$.

25. Express $\sin 3\theta$ in terms of powers of $\cos\theta$ and $\sin\theta$.

26. Solve the hyperbolic equation $\cosh x - 5\sinh x - 5 = 0$.

27. Four non-coplanar points *A, B, C, D* are positioned such that the line *AD* is perpendicular to *BC* and *BD* is perpendicular to *AC*. Show that *CD* is perpendicular to *AB*.

28. A point *P* divides a line segment *AB* in the ratio 2:3. If the position vectors of the points *A* and B are $\vec{a}$ and $\vec{b}$, respectively, find the position vector of the point *P*.

29. Find the surface area of a cone formed by rotating about the *x*-axis the line $y = 2x$ between $x = 0$ and $x = 5$.

30. Evaluate $I = \int \frac{1}{x^2 + 4x + 7} dx$.

31. The equation in polar coordinates of an ellipse with semi-axes *a* and *b* is $\frac{1}{\rho^2} = \frac{\cos^2\phi}{a^2} + \frac{\sin^2\phi}{b^2}$. Find the area of the ellipse.

(6 × 4 = 24 Marks)

SECTION – D

Answer **any two** questions. **Each** question carries **15** marks.

32. (a) State and prove Mean Value theorem.

    (b) Find $\frac{dy}{dx}$ if $x = \frac{t-2}{t+2}$ and $y = \frac{2t}{t+1}$.

33. (a) Simplify the expression $z = i^{-2i}$ to a real quantity.

    (b) Express $\cosh^{-1} x$ in terms of logarithms.

34. (a) Find the distance from the point $\vec{P}$ with coordinates (1,2,3) to the plane that contains the points A, B and C having coordinates (0,1,0), (2,3,1) and (5, 7, 2).

    (b) A line is given by $\vec{r} = \vec{a} + \lambda\vec{b}$, where $a = 5\vec{i} + 7\vec{j} + 9\vec{k}$ and $\vec{b} = 4\vec{i} + 5\vec{j} + 6\vec{k}$. Find the coordinates of the point $\vec{P}$ at which the line intersects the plane $x + 2y + 3z = 6$.

35. (a) Find the volume of a cone enclosed by the *surface* formed by rotating about the x-axis the line $y = 2x$ between $x = 0$ and $x = 3$.

    (b) Find the length of the curve $y = x^{3/2}$ from $x = 0$ to $x = 5$.

    (c) Evaluate $I = \int \frac{1}{x^3 + x}$.

(2 × 15 = 30 Marks)
