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P – 1258

Reg. No. :

Name :

Second Semester B.Sc. Degree Examination, September 2022

First Degree Programme under CBCSS

Mathematics

Complementary Course for Physics

MM 1231.1 : MATHEMATICS II – APPLICATIONS OF CALCULUS AND VECTOR DIFFERENTIATION

(2021 Admission)

Time : 3 Hours

Max. Marks : 80

SECTION – I

All the first ten questions are compulsory. They carry 1 mark each.

1. Find the intervals on which $f(x) = x^2 - 4x + 3$ is increasing.
2. If $f''(a)$ exists and f has an inflection point at $x = a$, then $f''(a) = \underline{\hspace{2cm}}$
3. Determine where the graph of the function $f(x) = x + 2 \sin x$ is concave up.
4. Define critical point of a function.
5. State Mean value theorem.
6. What is the value of $\lim_{x \rightarrow 0} \frac{e^x - 1}{\sin x}$?

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7. Find the rectangular coordinates of the point P whose polar coordinates are given by $(r, \theta) = (6, 2\pi/3)$.
8. Identify the quadric surface $z = \frac{x^2}{4} + \frac{y^2}{25}$.
9. Find the gradient of $f(x, y) = x^2 + xy + z^2$.
10. Find $r'(t)$ if $r(t) = t^2i + e^tj - (2 \cos \pi t)k$.

(10 × 1 = 10 Marks)

SECTION – II

Answer any **eight** questions from among the questions 11 to 26. These questions carry **2** marks each.

11. Find the x -coordinates of all inflection points of $3x^4 - 4x^3$.
12. Find the critical points of $f(x) = 3x^{5/3} - 15x^{2/3}$.
13. Let $f(x) = x^2 + bx + c$. Find the values of b and c such that $f(1) = 5$ is an extreme value of f on $[0, 2]$. Is this value a maximum or minimum?
14. Find an interval $[a, b]$ on which $f(x) = x^4 + x^3 - x^2 + x - 2$ satisfies the hypotheses of Rolle's theorem.
15. Find all values of k and l such that $\lim_{x \rightarrow 0} \frac{k + \cos lx}{x^2} = -4$.
16. Find the total area between the curve $y = 1 - x^2$ and the x axis over the interval $[0, 2]$.
17. Find area of the surface that is generated by revolving the portion of the curve $y^2 = x$ from origin to the point where $x = 2$ about the x -axis.
18. Find the exact arc length of the curve $y = 3x^{3/2} - 1$ from $x = 0$ to $x = 1$.

19. Evaluate : $\int_2^a \int_2^b \frac{dy dx}{xy}$.

20. Evaluate : $\int_{-1}^2 \int_0^2 \int_0^1 (x^2 + y^2 + z^2) dx dy dz$.

21. Find the perimeter of the astroid $x^{2/3} + y^{2/3} = a^{2/3}$.

22. Find the length of the arc of the parabola $y^2 = 4ax$ from the vertex to the extremity of the latus rectum.

23. Find the area of the surface that is generated by revolving the curve $y = \sqrt{4 - x^2}$, $-1 \leq x \leq 1$.

24. Find the domain of $r(t)$ and the value of $r(t_0)$ if $r(t) = \sqrt{5t+1} i + t^2 j$; $t_0 = 1$.

25. Describe the graph of $r(t) = \langle 1+2t, -1+3t \rangle$.

26. If $f(x, y) = (1 + xy)^{3/2}$, find $D_u f(8, 1)$ for the unit vector $u = \frac{1}{\sqrt{2}} i + \frac{1}{\sqrt{2}} j$.

(8 × 2 = 16 Marks)

SECTION – III

Answer any **six** questions from among the questions 27 to 38. These questions carry **4** marks each.

27. Find the relative extrema of $f(x) = 3x^5 - 5x^3$.

28. Use cylindrical shells to find the volume of the solid generated when the region enclosed between $y = x^3 - 3x^2 + 2x$ over $[0, 1]$ is revolved about the y-axis.

29. Use double integral to find the area bounded by the x-axis, $y = 2x$ and $x + y = 1$.

30. Evaluate the triple integral $\iiint_G 12xy^2 z^3 dV$ over the rectangular box G defined by the inequalities $-1 \leq x \leq 2, 0 \leq y \leq 3, 0 \leq z \leq 2$.
31. Evaluate $\lim_{x \rightarrow \infty} \left(\frac{1}{x} - \frac{1}{e^x - 1} \right)$.
32. Evaluate $\iint xy dx dy$ over the positive quadrant bounded by $\frac{x}{a} + \frac{y}{b} = 1$.
33. Find the length of the curve $27y^2 = 4x^3$ from $(0, 0)$ to $(3, 2)$.
34. Find the area of the region that is enclosed between the curves $y = x^2$ and $y = x + 6$.
35. Use polar coordinates to evaluate $\int_{-1}^1 \int_0^{\sqrt{1-x^2}} (x^2 + y^2)^{3/2} dy dx$.
36. Using double integral, find the area of the region R bounded by $y = x$ and $y = x^2$ in the first quadrant.
37. If $r_1(t) = 2ti + 3t^2j + t^3k$ and $r_2(t) = t^4k$. Find $\frac{d}{dt}(r_1 \circ r_2)$ and $\frac{d}{dt}(r_1 \times r_2)$.
38. Find the directional derivative of $f(x, y, z) = x^2y - yz^3 + z$ at the point $(1, -2, 0)$ in the direction of the vector $a = 2i + j - 2k$.

(6 × 4 = 24 Marks)

SECTION – IV

Answer any **two** questions from among the questions 39 to 44. These question carry **15** marks.

39. (a) Show that the function $f(x) = \frac{1}{4}x^3 + 1$ satisfies the hypotheses of the Mean Value Theorem over the interval $[0, 2]$ and find all values of c in the interval $(0, 2)$.
- (b) An open box is to be made from a 16-inch by 30-inch piece of cardboard by cutting out squares of equal size from the four corners and bending up the sides. What size should the squares be to obtain a box with the largest volume?

40. (a) Use cylindrical shells to find the volume of the solid generated when the region enclosed between $y = x^3 - 3x^2 + 2x$ over $[0, 1]$ is revolved about the y -axis.
- (b) Find the area of the astroid $x^{2/3} + y^{2/3} = a^{2/3}$.
41. (a) Find the area of the surface that is generated by revolving the portion of the curve $y = x^2$ between $x = 1$ and $x = 2$ about the y -axis.
- (b) Find the volume of the solid that results when the region above the x -axis and below the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 (a > 0, b > 0)$ is revolved about the x -axis.
42. (a) Use double integration to find the area of the region R enclosed between the parabola $y = \frac{1}{2}x^2$ and the line $y = 2x$.
- (b) Use triple integration in cylindrical coordinates to evaluate
- $$\int_0^a \int_{-\sqrt{a^2-x^2}}^{\sqrt{a^2-x^2}} \int_0^{a^2-x^2-y^2} x^2 dz dy dx (a > 0).$$
43. (a) Using integration find the area of the triangular region whose sides have the equations $y = 2x + 1$, $y = 3x + 1$ and $x = 4$.
- (b) Find the volume of the solid that results when the region enclosed by the curves $f(x) = \sin x$ and $g(x) = \cos x$ is revolved about the x -axis from $x = 0$ to $x = \pi/4$.

44. If the electrical potential at a point (x, y) in the xy -plane is $V(x, y)$, then the electrical intensive vector at the point (x, y) is $E = -\nabla V(x, y)$. Suppose that $V(x, y) = e^{-2x} \cos 2y$.

(a) Find the electric intensity vector at $(\pi/4, 0)$.

(b) If $f(x, y, z) = 3x^2y - y^3z^2$, find the maximum value of direction derivative of f at the point $(1, -2, -1)$.

(2 × 15 = 30 Marks)