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Reg. No. : ....

Name : ....

Second Semester B.Sc. Degree Examination, September 2022

First Degree Programme Under CBCSS

Mathematics

Complementary Course for Chemistry/Polymer Chemistry

MM 1231.2 : Mathematics II

CALCULUS WITH APPLICATIONS IN CHEMISTRY – II

(2020 Admission)

Time : 3 Hours Max. Marks : 80

SECTION – I

Answer **all** questions. **Each** carries **1** mark.

1. Find $\frac{\delta z}{\delta y}$ if $z = x^4 \sin(xy^3)$

2. Describe a saddle point.

3. $\lim_{n\to\infty} (-1)^{n+1} \frac{1}{n} =$ ____________

4. $\lim_{n\to\infty} n\sqrt{n} =$

5. Define conditional convergence of an infinite series.

6. The unit tangent vector at $t = \frac{\pi}{2}$ on the curve $x = 3\cos t$, $y = 3\sin t$ for $0 \le t \le 2\pi$ is ____________

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7. If $\bar{r} = x\hat{i} + y\hat{j} + z\hat{k}$, then $\nabla.\bar{r} =$ ____________

8. Curl $\overline{F}$, if the vector field $\overline{F}$ is irrotational is ____________

9. If $x = r\cos\theta$, $y = r\sin\theta$ then $\frac{\delta(x,y)}{\delta(r,\theta)} =$ ____________

10. $\int_0^2\int_0^3 dxdy =$ ____________

(10 × 1 = 10 Marks)

SECTION – II

Answer **any eight** questions. **Each** carries **2** marks.

11. Let $f(x,y) = x^2y + 5y^3$ Find the slope of the surface $z = f(x,y)$ in the y-direction at the point (1,–2)

12. Show that $(y+z)dx + xdy + xdz$ is an exact differential.

13. Let $f(x,y) = \begin{cases} \frac{-xy}{x^2+y^2}, (x,y) \neq (0,0) \\ 0 \quad , (x,y) = (0,0) \end{cases}$ Show that $\frac{\partial f}{\partial x}$ exist at all points (x,y)

14. Find the total derivative of $f(x,y) = x^2 + 3xy$ with respect to x given that $y = \sin^{-1} x$

15. Evaluate the Sum $\sum_{n=1}^{50} \frac{1}{n(n+2)}$

16. Test the convergence of the series $\sum_{n=1}^{\infty} \left(\frac{1}{n}\right)^n$

17. Determine the convergence of the series $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n}$

18. Show that the series $\sum_{n=1}^{\infty} \frac{1}{(n!)^2}$ converges.

19. Find $\frac{d\bar{r}}{du}$ of $\bar{r}(u) = (\tan^{-1} u)\hat{i} + u\cos u\hat{j} + \sqrt{u}\hat{k}$

20. Find $\bar{r}(u)$ given that $r^{-1}(u)=3\hat{i}+2u\hat{j}$ and $\bar{r}(1)=2\hat{i}+5\hat{j}$

21. Find the laplacian of the scalar field $\phi=xy^2z^3$

22. If ϕ is a scalar field, show that curl(grad ϕ)=$\bar{0}$

23. Evaluate $\int_0^a \int_0^{\sqrt{a^2-y^2}} dxdy$

24. State pappu's theorems.

25. Find the centre of mass of the solid hemisphere bounded by the surfaces $x^2+y^2+z^2=a^2$ and the $xy-$plane, assuming that it has a uniform density ρ

26. Evaluate the integral $\iint_R \left(1+\sqrt{x^2+y^2}\right)dxdy$ where R is the region bounded by the circle $x^2+y^2=1$

(8 × 2 = 16 Marks)

SECTION – III

Answer **any six** questions. **Each** carries **4** marks.

27. Find the Taylor expansion, upto quadratic terms in $(x-2)$ and $(y-3)$ of $f(x,y)=ye^{xy}$ about the point $(2,3)$

28. Locate the stationary point of x^3+y^3-3axy and determine their nature.

29. Expand $e^x \sin y$ in power of x and y as far as terms of the third degree.

30. Sum the series $\sum_{n=0}^{\infty}\frac{\cos(n\theta)}{n!}$

31. Find the real value of x for which the series $\sum_{n=1}^{\infty}\frac{x^n}{n+1}$ in convergent

32. Determine the absolute convergence of the series $\sum_{n=1}^{\infty}\frac{(-n)^n}{n^{5/2}}$

33. Find the divergence and curl of the vector field $F(x,y,z)=x^2y\hat{i}+2y^3z\hat{j}+3z\hat{k}$

34. Find a unit vector normal to the surface $x^3+y^3+3xyz=3$ at the point $(1,2,-1)$

35. If $\bar{r}=x\hat{i}+y\hat{j}+2\hat{k}$ and $r=|\bar{r}|$, show that $\nabla\left(\frac{1}{r}\right)=\frac{-\bar{r}}{r^2}$

36. Find the volume of the tetrahedron bounded by the three co-ordinate surfaces $x=0, y=0$ and $z=0$ and the plane $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$

37. Evaluate $\iiint_R (x+y+z)dxdydz$ where R is $0 \le x \le 1, 1 \le y \le 2, 2 \le z \le 3$

38. Evaluate $\int_{-\infty}^{\infty} e^{-x^2} dx$

(6 × 4 = 24 Marks)

SECTION – IV

Answer **any two** questions. **Each** question carries **15** marks.

39. A rectangular box, which is open at the top, has a capacity of 256 cubic feet. Determine the dimensions of the box such that the least material is required for the construction of the box.

    Use Lagrange's method of multiplies to obtain the solution.

40. Find the volume of the largest rectangular parallelepiped that can be inscribed in the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$

41. Find the machaurin's series for $\cos x$ and hence show that $(\cos x)^{-2} = 1 + x^2 + \frac{2x^4}{3} + \ldots$ Deduce the first three terms is the maclaurin's series for $\tan x$.

42. Express the vector field $\bar{a} = yz\hat{i} - y\hat{j} + xz^2\hat{k}$ in cylindrical polar coordinates, and hence calculate its divergence. Show that the same result is obtained by evaluating the divergence in Cartesian coordinates.

43. Find an expression for a volume element in spherical polar co-ordinates, and hence calculate the moment of inertia about a diameter of a uniform sphere of radius a and mass M.

44. Find the volume integral of x^2y over the tetrahedral volume bounded by the planes $x=0, y=0, z=0$ and $x+y+z=1$

(2 × 15 = 30 Marks)