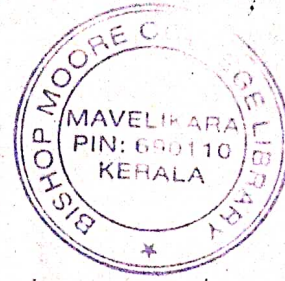


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P – 1234

Reg. No. :

Name :

Second Semester B.Sc. Degree Examination, September 2022

First Degree Programme under CBCSS

Mathematics

Foundation Course II

MM 1221 : FOUNDATIONS OF MATHEMATICS

(2018 & 2019 Admission)

Time : 3 Hours

Max. Marks : 80

SECTION – I

Answer all questions. Each question carries 1 mark. Answer in one word to a maximum of two sentences.

1. Find the range of the function $f(x) = 1 + \sin x$.
2. Prove that $p \rightarrow (p \vee q)$ is a tautology.
3. Give an example of a relation which is reflexive, transitive but not symmetric.
4. Symbolize the statement: Every degree student needs a course in Mathematics.
5. Write the negation of the statement: If he studies, he will pass the examination.
6. Find the rectangular co-ordinates of the point whose polar co-ordinates are given by $(r, \theta) = (4, \pi/6)$.
7. Identify the curve $r = \cos \theta$ by transforming to rectangular coordinates.

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8. State the reflection property of hyperbolas.
9. Identify the quadric surface $z = \frac{y^2}{b^2} - \frac{x^2}{a^2}$.
10. Find the direction cosine of the vector $v = 2i - 3j - k$.

(10 × 1 = 10 Marks)

SECTION – II

Answer any **eight** questions. Each question carries **2** marks.

11. Determine whether the following sentence is a statement: $x + 3$ is a positive integer
12. Write the negation of the statement: 9 is greater than 8 and 6 is less than 10
13. Find the truth value of the implication, if $0 + 0 = 0$ then $1 + 1 = 1$
14. Show that the statement $p \wedge (\sim p \wedge q)$ is a contradiction where p and q are primitive statements
15. If R is a relation on the set Z of integers defined by $x R y$ if and only if $x^2 = y^2$ Prove that R is an equivalence relation
16. Check whether the function defined by $f(x) = e^{x^2}$ is injective.
17. Find all values of t for the parametric curve $x = 2 \sin t, y = 4 \cos t (0 \leq t \leq 2\pi)$ where the slope of the tangent line is zero.
18. Find the equation of the hyperbola with vertices $(\pm 2, 0)$ and foci $(\pm 3, 0)$.
19. Find the new coordinates of the point $(-4, 2)$ if the coordinate axes are rotated through an angle of 60° .
20. Find the unit vector that has the same direction as $v = 2i + 2j - k$.
21. Sketch the graph of $x^2 + z^2 = 1$ in 3-space.
22. Find the direction cosine of the vector $v = 2i - 4j + 2k$.

(8 × 2 = 16 Marks)

SECTION – III

Answer any **six** questions. Each question carries **4** marks.

23. Construct the truth table for $[p \wedge (p \Rightarrow q) \Rightarrow q]$
24. Let $A = \{1, 2, 3, 4, 5\}$. Consider the relation R on A defined as $R = \{(1,1), (2,2), (3,3), (4,4), (5,5), (1,5), (5,1), (5,3), (3,5)\}$. Is R an equivalence relation?
25. Let $f: A \rightarrow B$ and $B \rightarrow C$ are surjective, show that $g \circ f$ is surjective
26. Find the total arc length of the cardioid $r = 1 + \cos \theta$
27. Find the entire area of the region that is inside of the cardioid $r = 4 + 4 \cos \theta$ and outside of the circle $r = 6$
28. Describe the graph of the equation $x^2 - y^2 - 4x + 8y - 21 = 0$
29. (a) Find the vector of length 2 that has an angle of $\pi/4$ with the positive x-axis
(b) Find the angle that the vector makes with the positive x-axis
30. Suppose that the axes of an xy -coordinate system are rotated through an angle of 45° to obtain an $x'y'$ -coordinate system. Find the equation of the curve $x^2 - xy + y^2 - 6 = 0$ in $x'y'$ -coordinates
31. Find the parametric equation of the line
 - (a) Passing through $(1, 2, -3)$ and parallel to $v = 4i + 5j - 7k$
 - (b) Passing through the origin in 3-space and parallel to $v = \langle 1, 1, 1 \rangle$

(6 × 4 = 24 Marks)

SECTION – IV

Answer any **two** questions. Each question carries **15** marks.

32. The relation R on the set of integers Z is defined by $x R y$ if and only if $x - y = 2k$ for some integer k .
 - (a) Verify that R is an equivalence relation on Z
 - (b) Determine the equivalent classes and a partition of Z induced by R .

(a) Determine the truth value of the following statements if the universe comprises all non zero integers

(i) $\exists x \exists y$ such that $xy=2$

(ii) $\forall x \exists y$ such that $xy=2$

(b) Let $f:A \rightarrow B$ and $g:B \rightarrow C$ be invertible functions. Show that the composition $g \circ f:A \rightarrow C$ is also invertible and $(g \circ f)^{-1} = f^{-1} \circ g^{-1}$.

(a) Identify and sketch the curve $xy=1$.

(b) Sketch the graph of the following equations in polar coordinates

(i) $r=1$

(ii) $\theta=\pi/3$

(iii) $r=\theta(\theta \geq 0)$

(a) Find the equation of the plane through the points $P_1(-2,1,1), P_2(0,2,3)$ and $P_3(1,0,-1)$.

(b) Let L_1 and L_2 given by

$$L_1: x=1+4t, y=5-4t, z=-1+5t \text{ and}$$

$$L_2: x=2+8t, y=4-3t, z=5+t \text{ be two lines.}$$

(i) Are the lines parallel?

(ii) Do the lines intersect?

(2 × 15 = 30 Marks)