(Pages : 4)

P – 1234

Reg. No. : ....

Name : ....

Second Semester B.Sc. Degree Examination, September 2022

First Degree Programme under CBCSS

Mathematics

Foundation Course II

MM 1221 : FOUNDATIONS OF MATHEMATICS

(2018 & 2019 Admission)

Time : 3 Hours

Max. Marks : 80

SECTION – I

Answer **all** questions. Each question carries **1** mark. Answer in one word to a maximum of two sentences.

1. Find the range of the function $f(x) = 1 + \sin x$.

2. Prove that $p \to (p \vee q)$ is a tautology.

3. Give an example of a relation which is reflexive, transitive but not symmetric.

4. Symbolize the statement: Every degree student needs a course in Mathematics.

5. Write the negation of the statement: If he studies, he will pass the examination.

6. Find the rectangular co-ordinates of the point whose polar co-ordinates are given by $(r, \theta) = (4, \pi/6)$.

7. Identify the curve $r = \cos\theta$ by transforming to rectangular coordinates.

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8. State the reflection property of hyperbolas.

9. Identify the quadric surface $z=\frac{y^2}{b^2}-\frac{x^2}{a^2}$.

10. Find the direction cosine of the vector $v=2i-3j-k$.

(10 × 1 = 10 Marks)

SECTION – II

Answer any **eight** questions. Each question carries **2** marks.

11. Determine whether the following sentence is a statement: $x+3$ is a positive integer

12. Write the negation of the statement: 9 is greater than 8 and 6 is less than 10

13. Find the truth value of the implication, if 0 + 0 = 0 then 1 + 1 = 1

14. Show that the statement $p\wedge(\sim p\wedge q)$ is a contradiction where p and q are primitive statements

15. If R is a relation on the set Z of integers defined by $x\,R\,y$ if and only if $x^2=y^2$ Prove that R is an equivalence relation

16. Check whether the function defined by $f(x)=e^{x^2}$ is injective.

17. Find all values of t for the parametric curve $x=2\sin t, y=4\cos t (0\le t\le 2\pi)$ where the slope of the tangent line in zero.

18. Find the equation of the hyperbola with vertices (± 2,0) and foci (± 3,0).

19. Find the new coordinates of the point (– 4,2) if the coordinate axes are rotated through an angle of 60°.

20. Find the unit vector that has the same direction as $v=2i+2j-k$.

21. Sketch the graph of $x^2+z^2=1$ in 3-space.

22. Find the direction cosine of the vector $v=2i-4j+2k$.

(8 × 2 = 16 Marks)

SECTION – III

Answer any **six** questions. Each question carries **4** marks.

23. Construct the truth table for $[p \wedge (p \Rightarrow q) \Rightarrow q]$

24. Let A = {1, 2.3. 4,5}. Consider the relation R on A defined as R = {(1,1), (2,2), (3,3), (4.4), (5,5), (1,5), (5,1), (5,3), (3,5)). Is R an equivalence relation?

25. Let $f: A \to B$ and $B \to C$ are surjective, show that $g \circ f$ is surjective

26. Find the total arc length of the cardioid $r = 1 + \cos\theta$

27. Find the entire area of the region that is inside of the cardioid $r = 4 + 4\cos\theta$ and outside of the circle $r = 6$

28. Describe the graph of the equation $x^2 - y^2 - 4x + 8y - 21 = 0$

29. (a) Find the vector of length 2 that has an angle of $\pi/4$ with the positive x-axis

    (b) Find the angle that the vector makes with the positive x-axis

30. Suppose that the axes of an xy- coordinate system are rotated through an angle of 45° to obtain an $x'y'$- coordinate system. Find the equation of the curve $x^2 - xy + y^2 - 6 = 0$ in $x'y'$-coordinates

31. Find the parametric equation of the line

    (a) Passing through (1,2, –3) and parallel to $v = 4i + 5j - 7k$

    (b) Passing through the origin in 3-space and parallel to $v = \langle 1,1,1 \rangle$

(6 × 4 = 24 Marks)

SECTION – IV

Answer any **two** questions. **Each** question carries **15** marks.

32. The relation R on the set of integers Z is defined by $x\ R\ y$ if and only if $x - y = 2k$ for some integer k.

    (a) Verify that R is an equivalence relation on Z

    (b) Determine the equivalent classes and a partition of Z induced by R

(a) Determine the truth value of the following statements if the universe comprises all non zero integers

(i) $\exists x \exists y$ such that $xy = 2$

(ii) $\forall x \exists y$ such that $xy = 2$

(b) Let $f : A \to B$ and $g : B \to C$ be invertible functions. Show that the composition $g \circ f : A \to C$ is also invertible and $(g \circ f)^{-1} = f^{-1} \circ g^{-1}$.

(a) Identify and sketch the curve $xy = 1$.

(b) Sketch the graph of the following equations in polar coordinates

(i) $r = 1$

(ii) $\theta = \pi / 3$

(iii) $r = \theta (\theta \geq 0)$

(a) Find the equation of the plane through the points $P_1(-2,1,1), P_2(0,2,3)$ and $P_3(1,0,-1)$.

(b) Let L_1 and L_2 given by

$L_1 : x = 1 + 4t, y = 5 - 4t, z = -1 + 5t$ and

$L_2 : x = 2 + 8t, y = 4 - 3t, z = 5 + t$ be two lines.

(i) Are the lines parallel?

(ii) Do the lines intersect?

(2 × 15 = 30 Marks)
