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P – 1272

Reg. No. : ....

Name : ....

Second Semester B.Sc. Degree Examination, September 2022

First Degree Programme under CBCSS

Mathematics

Complementary Course for Chemistry/Polymer Chemistry

MM 1231.2 : MATHEMATICS II — INTEGRAL CALCULUS AND VECTOR DIFFERENTIATION

(2021 Admission)

Time : Three Hours

Max. Marks : 80

PART – A

Answer **all** questions :

1. Evaluate : $\int \tan^2 x \, dx$

2. Estimate : $\int_0^{2\pi} \cos x \, dx$

3. Find : $\frac{d}{dx}\int_1^x t^3 \, dt$

4. Give an example of a solid of revolution.

5. Write the first – order model of the period T of a simple pendulum.

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6. Define a cardioid.

7. Let T be the transformation from the uv - plane to the xy - plane defined by the equations $x=\frac{1}{4}(u+v)$, $y=\frac{1}{2}(u-v)$. Find $T(1,3)$.

8. Determine $\lim_{t\to 3}\left(t^2 i+2t\, j\right)$.

9. Define the directional derivative of f in the direction of u at (x_0, y_0, z_0).

10. Compute $\int_0^1 r(t)dt$, where $r(t)=t^2 i+e^t j-(2\cos\pi t)k$.

(10 × 1 = 10 Marks)

PART – B

Answer **any eight** questions.

11. Evaluate : $\int e^{\tan x}\sec^2 x\, dx$.

12. Find the area under the curve $y=\cos x$ over the interval $[0, \pi/2]$.

13. State the mean - value theorem for integrals.

14. Evaluate : $\int \frac{dx}{\sqrt{2-x^2}}$.

15. Using integration by parts, evaluate : $\int \left(x^2-x\right)\cos x\, dx$.

16. Compute : $\int_0^{\pi/4}\cos x\, dx+\int_{\pi/4}^{\pi/2}\cos x\, dx+\int_{\pi/2}^{\pi}\cos x\, dx$.

17. Find the area of the region bounded above by $y=x+6$, bounded below by $y=x^2$, and bounded on the sides by the lines $x=0$ and $x=2$.

18. Differentiate the Bessel function $J_0(x)$ with respect to x.

19. Write the conversion formulas for cylindrical coordinate system to rectangular coordinate system.

20. Define the Jacobian of the transformation T from the uv - plane to the xy -plane defined by the equations $x=x(u,v), y=y(u,v)$.

21. Find the rectangular coordinates of the point whose polar coordinates are $\left(6, \frac{2\pi}{3}\right)$.

22. Prove that the graph $r=\cos 2\theta$ is symmetric about the x-axis and y-axis.

23. Verify whether $\int_0^1\int_0^1 xy^2\,dx\,dy = \int_0^1\int_0^1 xy^2\,dy\,dx$.

24. Find the natural domain of $r(t)=\left\langle \ln|t-1|, e^t, \sqrt{t}\right\rangle$.

25. Let $f(x, y)=x^2e^y$. Estimate the maximum value of a directional derivative at $(-2, 0)$ and find the unit vector in the direction in which the maximum value occurs.

26. If $r'(t)=(3, 2t)$ and $r(1)=(2,5)$, then find $r(t)$.

(8 × 2 = 16 Marks)

PART – C

Answer **any six** questions.

27. Evaluate : $\int \frac{dx}{x^2+x-2}$

28. Compute the value of the integral $\int_0^3 f(x)dx$ where

$$f(x)=\begin{cases} x^2, & x<2 \\ 3x-2, & x\geq 2 \end{cases}$$

29. Find : $\int x^2\sqrt{x-1}\,dx$.

30. Derive the formula for the volume of a sphere of radius *r*.

31. Find the first three nonzero terms in the Maclaurin series for $\tan x$.

32. Estimate : $\int_0^2\int_{y/2}^1 e^{x^2}\,dx\,dy$.

33. Use a polar double integral to find the area enclosed by the three — petaled rose $r=\sin 3\theta$.

34. Derive the equation of the tangent plane to the parametric surface $x=uv$, $y=u$, $z=v^2$ at the point where $u=2$ and $v=-1$.

35. Estimate $\iiint_G 12xy^2z^3\,dV$ over the rectangular box G defined by the inequalities $-1\leq x\leq 2$, $0\leq y\leq 3$, $0\leq z\leq 2$.

36. Evaluate $\iint_R y^2x\,dA$ over the rectangle $R=\{(x,y):-3\leq x\leq 2, 0\leq y\leq 1\}$.

37. Estimate $\int_2^4\int_1^3 (40-2xy)\,dx\,dy$.

38. Let $r_1(t)=\left(\tan^{-1}t\right)i+(\sin t)j+t^2k$ and $r_2(t)=\left(t^2-t\right)i+(2t-2)j+(\ln t)k$. Compute the degree measure of the acute angle between the tangent lines to the graphs of $r_1(t)$ and $r_2(t)$ at the origin.

(6 × 4 = 24 Marks)

PART – D

Answer **any two** questions.

39. Evaluate : $\int \frac{x^2+x-2}{3x^3-x^2+3x-1}dx$.

40. Evaluate : (a) $\int_0^{3/4}\frac{dx}{1-x}$ (b) $\int_0^{\ln 3} e^x\left(1+e^x\right)^{1/2}dx$ (c) $\int_0^{\pi/8}\sin^5 2x\cos 2x\,dx$

(d) $\int_2^5 (2x-5)(x-3)^9\,dx$.

41. (a) Estimate the area of the surface that is generated by revolving the portion of the curve $y=x^3$ between $x=0$ and $x=1$ about the x-axis.

(b) Compute the arc length of the curve $y=x^{3/2}$ from $(1,1)$ to $\left(2,2,\sqrt{2}\right)$.

42. Sketch the graph $r^2=4\cos 2\theta$ in polar coordinates.

43. (a) Use cylindrical coordinates to compute $\int_{-3}^{3}\int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}}\int_{0}^{9-x^2-y^2} x^2\, dz\, dy\, dx$.

(b) Find the surface area of the portion of the paraboloid $z=x^2+y^2$ below the plane $z=1$.

44. (a) Find the volume of the region enclosed by the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$.

(b) Evaluate : $\iint_R xy\,dA$ over the region R enclosed between $y=\frac{x}{2}$, $y=\sqrt{x}$, $x=2$ and $x=4$.

(2 × 15 = 30 Marks)
