(Pages : 6)

P – 1248

Reg. No. : ....

Name : ....

Second Semester B.Sc. Degree Examination, September 2022

First Degree Programme under CBCSS

Mathematics

Complementary Course for Physics

MM 1231.1 : MATHEMATICS-II-INTEGRATION AND VECTORS

(2014-2017 Admissions)

Time : 3 Hours

Max. Marks : 80

SECTION – I

All the first **ten** questions are compulsory. They carry **1** mark each.

1. For the function $f = 8$, find the area $A(x)$ between the graph of f and the interval $[a, x] = [-1, x]$, and find the derivative $A'(x)$ of this area function.

2. Find $\int \frac{t^2 - 2t^4}{t^4} dt$.

3. True or False : "If the particle has constant acceleration, the velocity versus time graph will be a straight line."

4. Suppose that a particle moves along a coordinate line so that its velocity at time t is $v(t) = 2 + \cos t$. Find the average velocity of the particle during the time interval $0 \le t \le \pi$.

P.T.O.

5. Evaluate $\int_{1}^{3}\int_{2}^{4}(40-2xy)\,dydx$.

6. Determine whether $r(t)=r(t)=\cos t^2 i+\sin t^2 j+e^t k$ is a smooth function of the parameter t.

7. Define a conservative field and potential function.

8. Evaluate $\int_C F.dr$ along the line segment C from P to Q; where $F(x, y)=8i+8j$; $P(-4, 4), Q(-4, 5)$.

9. Find the directional derivative of $f(x, y)=4x^3y^2$ at $P=(2, 1)$ in the direction of $a=4i-3j$.

10. Find the value of curl f, if $f=x^2i+4xy^3j+y^2xk$.

(10 × 1 = 10 Marks)

SECTION – II

Answer **any eight** questions. These questions carry **2** marks each.

11. Find the total area between the curve $y=1-x^2$ and the x – axis over the interval [0, 2].

12. A particle moves along an s-axis. Use the information $v(t)=3t^2-2t$; $s(0)=1$ to find the position function of the particle.

13. Derive the formula for the volume of a sphere of radius r.

14. Find the area of the surface generated by revolving the curve $y = 7x; 0 \le x \le 1$ about the x-axis.

15. Evaluate $\int_{1}^{2}\int_{0}^{y^2} e^{x/y^2}\,dxdy$.

16. Evaluate the triple integral $\int_{-1}^{1}\int_{0}^{2}\int_{0}^{1}(x^2 + y^2 + z^2)\,dx\,dy\,dz$.

17. Find the gradient of $f(x, y) = 5x^2 + y^4$ at the indicated point (4, 2).

18. Find an equation for the tangent plane and parametric equations for the normal line to the surface $x^2y^4z^2 = -7$ at the point $P(-3, 1, -2)$.

19. If $F(x, y, z) = x^2i - 2j + yzk$, then find $div\ F$ and $curl\ F$.

20. Evaluate the line integral $\int_C (1 + xy^2)\,ds$, if $C : r(t) = ti + 2tj$ where $0 \le t \le 1$.

21. Let $F(x, y) = 2xy^3i + (1 + 3x^2y^2)j$. Show that F is conservative vector field on the entire xy-plane.

22. Use Greens Theorem to evaluate the integral $\oint 3xydx + 2xydy$, where C is the rectangle bounded by $x = -2$, $x = 4$, $y = 1$ and $y = 2$.

(8 × 2 = 16 Marks)

SECTION – III

Answer **any six** questions. These questions carry **4** marks each.

23. The temperature of a 10 m long metal bar is 15°C at one end and 30°C at the other end. Assuming that the temperature increases linearly from the cooler end to the hotter end, what is the average temperature of the bar?

24. Derive the formula for the volume of a right pyramid whose altitude is h and whose base is a square with sides of length a.

25. Let V be the volume of the solid that results when the region enclosed by $y = 1/x$, $y = 0$, $x = 2$, and $x = b(0 < b < 2)$ is revolved about the x-axis. Find the value of b for which $V = 3$.

26. Find the area of the surface that is generated by revolving the portion of the curve $y = x^3$ between $x = 0$ and $x = 1$ about the x-axis.

27. A circular lens of radius 2 inches has thickness $(1-(r^2/4))$ inches at all points r inches from the center of the lens. Find the average thickness of the lens.

28. Find parametric equations for the tangent line to the curve of intersection of the paraboloid $z = x^2 + y^2$ and the ellipsoid $x^2 + 4y^2 + z^2 = 9$, at the point (1, – 1, 2).

29. Show that the divergence of the inverse-square field

$$F(x, y, z) = \frac{c}{(x^2 + y^2 + z^2)^{3/2}} (x_i + y_j + z_k) \text{ is zero.}$$

30. Evaluate the line integral $\int_C (xy + z^3)ds$ from $(1, 0, 0)$ to $(-1, 0, \pi)$ along the helix C that is represented by the parametric equation.

$x = \cos t$ $y = \sin t$ $z = t$ $(0 \leq t \leq \pi)$

31. Confirm that the force field $F(x, y) = xy^2 i + x^2 yj$ is conservative in some open connected region containing the points $P(1, 1)$ and $Q(0, 0)$, and then find the work done by the force field on a particle moving along an arbitrary smooth curve in the region from P to Q.

(6 × 4 = 24 Marks)

SECTION – IV

Answer **any two** questions. These questions carry **15** marks each.

32. (a) Find the area of the region enclosed by $x = y^2$ and $y = x - 2$, integrating with respect to y.

(b) Use a double integral to find the volume of the solid that is bounded above by the plane $z = 4 - x - y$ and below by the rectangle $R = [0, 1] \times [0, 2]$.

33. (a) A rock is thrown downward from the top of building, 168 ft high, at an angle of 60° with the horizontal. How far from the basic of the building will the rock land if its initial speed is 80 ft / s?

(b) Find the directional derivative of $f(x, y) = e^{xy}$ at $(-2, 0)$ in the direction of the unit vector that makes an angle of $\pi/3$ with the positive x-axis.

34. (a) Use a line integral to find the area enclosed by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

(b) Use the Divergence Theorem to find the outward flux of the vector field

$F(x, y, z) = 2xi + 3yi + z^2 k$

across the unit cube.

35. (a) Use Greens theorem to evaluate the integral $\oint (x^2 - y^2)dx + xdy$, where C is the circle $x^2 + y^2 = 9$.

(b) Find the work performed by the force field $F(x, y, z) = x^2 i + 4xy^3 j + y^2 xkz$ on a particle that traverses the rectangle C in the plane $z = y$.

(2 × 15 = 30 Marks)
